The cost of installing insulation in a particular two-bedroom house is $2400. Present monthly heating bills average $60, but the insulation is expected to reduce the heating cost by 10%. How many months will it take to recover the cost of the insulation?

Municipal funding City government has approved the construction of a $50 million sports arena. Up to $30 million will be raised by selling bonds that pay simple interest at a rate of 12% annually. The remaining amount (up to $40 million) will be obtained by borrowing money from an insurance company at a simple interest rate of 10%. Determine whether the arena can be financed so that the annual interest is $5.2 million.

Walking rates Two children who are 224 meters apart, start walking toward each other at the same instant at rates of 1.5m/sec and 2 m/sec, respectively.
(a) When will they meet?
(b) How far will each have walked?

In a certain medical test designed to measure carbohydrate tolerance, and adult drinks 7 ounces of a 30% glucose solution. When the test is administered to a child, the glucose concentration must be decreased to 20%. How much 30% glucose solution and how much water should be used to prepare 7 ounces of 20% glucose solution?

Theophylline, is prepared from an elixir with a drug concentration of 5 mg/ml and a cherry-flavored syrup that is to be added to hide the taste of the drug. How much of each must be used to prepare 100 milliliters of solution with a drug concentration of 2 mg/ml?

A wafer cone is to hold 8 in^3 of ice cream when filled to the bottom. The diameter of the cone is 2 inches, and the top of the ice cream has the shape of a hemisphere. Find the height (h) of the cone.

A runner starts at the beginning of a runners' path and runs at a constant rate of 6 mi/hr. Five minutes later a second runner begins at the same point, running at a rate of 8 mi/hr and following the same course. How long will it take the second runner to reach the first?

A farmer plans to use 180 feet of fencing to enclose a rectangular region, using part of a straight river bank instead of fencing as one side. Find the area of the region if the length of the side parallel to the river bank is
a. twice the length of the opposite side
b. one-half the length of the opposite side
c. the same length of an adjacent side

A bullet is fired horizontally at a target, and the sound of its impact is heard 1.5 seconds later. If the speed is 3300 ft/sec and the speed of sound is 1100 ft/sec, how far away is the target?

A woman begins jogging at 3:00 P.M., running due north at a 6- minute- mile pace. Later, she reverses direction and runs due south at a 7- minute mile pace. If she returns to her starting point at 3:45 P.M., find the total number of miles run.

Every cross section of a drainage ditch is an isosceles trapezoid with a small base of 3 feet and a height of 1 foot. Determine the width of the larger base that would give the ditch a cross-sectional area of $5ft^2$.

A large grain silo is to be constructed in the shape of a circular cylinder with a hemisphere attached to the top (see the figure). The diameter of the silo is to be 30 feet, but the height is yet to be determined. Find the height h of the silo that will result in a capacity of 11250π ft^3

Below the cloud base, the air temperature T (in F) at height h (in feet) can be approximated by the equation $T = T_0 - (5.5/1000)\,h$, where T_o is the temperature at ground level.
(a) Determine the air temperature at a height of 1 mile if the ground temperature is 70 ^oF.
(b) At what altitude is the temperature freezing?

An airplane flying north at 200 mph passed over a point on the ground at 2:00 PM. Another airplane at the same altitude passed over the point at 2:30 PM, flying east at 400 mph.
(a) If t denotes the time in hours after 2:30 P.M., express the distance d between the airplanes in terms of t.
(b) At what time were the planes 500 miles apart?

When a rock is dropped from a cliff into an ocean, it travels approximately $16t^2$ feet in t seconds. If the splash is heard 4 seconds later and the speed of sound is 1100 ft/sec, approximate the height of the cliff.

When a popular brand of CD player is priced at $300 per unit, a store sells 15 units per week. Each time the price is reduced by $10, however, the sales increase by 2 per week. What selling price will result in weekly revenues of $7000?

A closed right circular cylindrical oil drum of height 4 feet is to be constructed so that the total surface area is 10π ft^2. Find the diameter of the drum.

The rate at which a tablet of vitamin C begins to dissolve depends on the surface area of the tablet. One brand of tablet is 2 centimeters long and is in the shape of a cylinder with hemispheres of diameter 0.5 centimeter attached to both ends, as shown in the figure. A second brand of tablet is to be manufactured in the shape of a right circular cylinder of altitude 0.5 centimeter.
(a) Find the diameter of the second tablet so that its surface area is equal to that of the first tablet.
(b) Find the volume of each tablet.

The *withdrawal resistance* of a nail indicates its holding strength in wood. A formula that is used for bright common nails is $P = 15700S^{5/2}RD$, where P is the maximum withdrawal resistance (in pounds), S is the specific gravity of the wood at 12% moisture content, R is the radius of the nail (in inches), and D is the depth (in inches) that the nail has penetrated the wood. A 6d (sixpenny) bright, common nail of length 2 inches and diameter 0.113 inch is driven completely into a piece of Douglas fir. If it requires a maximum force of 380 pounds to remove the nail, approximate the specific gravity of Douglas fir.

Ladder height The recommended distance d that a ladder should be placed away from a vertical wall is 25% of its length L. Approximate the height h that can be reached by relating h as a percentage of L.

The urban heat island Urban areas have higher average air temperatures than rural areas, as a result of the presence of buildings, asphalt, and concrete. This phenomenon has become known as the *urban heat island.* The temperature difference T (in _C) between urban and rural areas near Montreal, with a population P between 1000 and 1,000,000, can be described by the formula $T = 0.25P^{1/4}/\sqrt{v}$, where v is the average wind speed (in mi/hr) and $v \geq 1$. If $T = 3$ and $v = 5$, find P.

Installing a power line A power line is to be installed across a river that is 1 mile wide to a town that is 5 miles downstream (see the figure). It costs $7500 per mile to lay the cable underwater and $6000 per mile to lay it overland. Determine how the cable should be installed if $35,000 has been allocated for this project.

Volume of a box From a rectangular piece of metal having dimensions 24 inches by 36 inches, an open box is to be made by cutting out an identical square of area x^2 from each corner and turning up the sides.
(a) Determine an equation for the volume V of the box in terms of x.
(b) Use a table utility to approximate the value of x within inch that will produce the maximum volume.

Constructing a box A cardboard box with an open top and a square bottom is to have a volume of 25 ft^3. Use a table utility to determine the dimensions of the box to the nearest 0.1 foot that will minimize the amount of cardboard used to construct the box.

Linear magnification Shown in the figure is a simple magnifier consisting of a convex lens. The object to be magnified is positioned so that the distance p from the lens is less than the focal length f. The linear magnification M is the ratio of the image size to the object size. It is shown in physics that $M = f/(f - p)$. If $f = 6$ cm, how far should the object be placed from the lens so that its image appears at least three times as large? (Compare with Example 4.)

Drug concentration To treat arrhythmia (irregular heartbeat), a drug is fed intravenously into the bloodstream. Suppose that the concentration c of the drug after t hours is given by $c = 3.5t/(t + 1)$ mg/L. If the minimum therapeutic level is 1.5 mg/L, determine when this level is exceeded.

Business expenditure A construction firm is trying to decide which of two models of a crane to purchase. Model A costs $100,000 and requires $8000 per year to maintain. Model B has an initial cost of $80,000 and a maintenance cost of $11,000 per year. For how many years must model A be used before it becomes more economical than B?

Buying a car A consumer is trying to decide whether to purchase car A or car B. Car A costs $20,000 and has an mpg rating of 30, and insurance is $1000 per year. Car B costs $24,000 and has an mpg rating of 50, and insurance is $1200 per year. Assume that the consumer drives 15,000 miles per year and that the price of gas remains constant at $3 per gallon. Based only on these facts, determine how long it will take for the total cost of car B to become less than that of car A.

Vertical leap record *Guinness Book of World Records* reports that German shepherds can make vertical leaps of over 10 feet when scaling walls. If the distance s (in feet) off the ground after t seconds is given by the equation $s = -16t^2 + 24t + 1$, for how many seconds is the dog more than 9 feet off the ground?

Height of a projected object If an object is projected vertically upward from ground level with an initial velocity of 320 ft/s, then its distance s above the ground after t seconds is given by $s = -16t^2 + 320t$. For what values of t will the object be more than 1536 feet above the ground?

Braking distance The braking distance d (in feet) of a certain car traveling is given by the equation $d = v + (v^2/20)$. Determine the velocities that result in braking distances of less than 75 feet.

Gas mileage The number of miles M that a certain compact car can travel on 1 gallon of gasoline is related to its speed v(in mi/hr) by

$$M = -\frac{1}{30}v^2 + \frac{5}{2}v \qquad 0 < v < 70$$

For what speeds will M be at least 45?

Decreasing height A person's height will typically decrease by 0.024 inch each year after age 30.

(a) If a woman was 5 feet 9 inches tall at age 30, predict her height at age 70.
(b) A 50-year-old man is 5 feet 6 inches tall. Determine an inequality for the range of heights (in inches) that this man will experience between the ages of 30 and 70.

Aircraft's landing speed In the design of certain small turboprop aircraft, the landing speed V (in ft/s) is determined by the formula $W = 0.00334V^2S$, where W is the gross weight (in pounds) of the aircraft and S is the surface area (in ft^2) of the wings. If the gross weight of the aircraft is between 7500 pounds and 10,000 pounds and $S = 210\ ft^2$, determine the range of the landing speeds in miles per hour.

Investment income An investor has a choice of two investments: a bond fund and a stock fund. The bond fund yields 7.186% interest annually, which is nontaxable at both the federal and state levels. Suppose the investor pays federal income tax at a rate of 28% and state income tax at a rate of 7%. Determine what the annual yield must be on the taxable stock fund so that the two funds pay the same amount of net interest income to the investor.

Gold and silver mixture A ring that weighs 80 grams is made of gold and silver. By measuring the displacement of the ring in water, it has been determined that the ring has a volume of 5 cm^3. Gold weighs 19.3 g/cm^3, and silver weighs 10.5 g/cm^3. How many grams of gold does the ring contain?

Preparing hospital food A hospital dietitian wishes to prepare a 10-ounce meat-vegetable dish that will provide 7 grams of protein. If an ounce of the vegetable portion supplies $\frac{1}{2}$ gram of protein and an ounce of meat supplies 1 gram of protein, how much of each should be used?

Solar heating A large solar heating panel requires 120 gallons of a fluid that is 30% antifreeze. The fluid comes in either a 50% solution or a 20% solution. How many gallons of each should be used to prepare the 120-gallon solution?

Passing speed An automobile 20 feet long overtakes a truck 40 feet long that is traveling at 50 mi/hr (see the figure). At what constant speed must the automobile travel in order to pass the truck in 5 seconds?

Filling a bin An extruder can fill an empty bin in 2 hours, and a packaging crew can empty a full bin in 5 hours. If a bin is half full when an extruder begins to fill it and a crew begins to empty it, how long will it take to fill the bin?

Highway travel A north-south highway intersects an eastwest highway at a point P. An automobile crosses P at 10 A.M., traveling east at a constant rate of 20 mi/hr. At the same instant another automobile is 2 miles north of P, traveling south at 50 mi/hr.
(a) Find a formula for the distance d between the automobiles t hours after 10:00 A.M.
(b) At approximately what time will the automobiles be 104 miles apart?

Fencing a kennel A kennel owner has 270 feet of fencing material to be used to divide a rectangular area into 10 equal pens, as shown in the figure. Find dimensions that would allow 100 ft^2 for each pen.

Dimensions of an aquarium An open-topped aquarium is to be constructed with 6-foot-long sides and square ends, as shown in the figure.
(a) Find the height of the aquarium if the volume is to be 48 ft^3.
(b) Find the height if 44 ft^2 of glass is to be used.

Boyle's law Boyle's law for a certain gas states that if the temperature is constant, then pv = 200, where p is the pressure (in lb/in^2) and v is the volume (in in^3). If $25 \le v \le 50$, what is the corresponding range for p?

Sales commission A recent college graduate has job offers for a sales position in two computer firms. Job A pays $50,000 per year plus 10% commission. Job B pays only $40,000 per year, but the commission rate is 20%. How much yearly business must the salesman do for the second job to be more lucrative?

Speed of sound The speed of sound in air at 0°C (or 273 K) is 1087 ft/sec, but this speed increases as the temperature rises. The speed v of sound at temperature T in K is given by $v = 1087\sqrt{T/273}$. At what temperatures does the speed of sound exceed 1100 ft/sec?

Planting an apple orchard The owner of an apple orchard estimates that if 24 trees are planted per acre, then each mature tree will yield 600 apples per year. For each additional tree planted per acre, the number of apples produced by each tree decreases by 12 per year. How many trees should be planted per acre to obtain at least 16,416 apples per year?

Apartment rentals A real estate company owns 218 efficiency apartments, which are fully occupied when the rent is $940 per month. The company estimates that for each $25 increase in rent, 5 apartments will become unoccupied. What rent should be charged in order to pay the monthly bills, which total $205,920?

(a) Find the distance d(A,B) between the points A(-1,0) and B(4,3).
(b) Find the midpoint of the segment AB.

Show that the triangle with vertices A(6,-7), B(11,-3), and C(2,-2) is a right triangle, and find its area.

Show that A(-4,2), B(1,4), C (3,-1) and D(-2,-3) are vertices of a square.

Show that A(-4,-1), B(0,-2), C (6,1) and D(2,2) are vertices of a parallelogram.

Find a formula that expresses the fact that an arbitrary point $P(x, y)$ is on the perpendicular bisector l of segment AB: $A(-4, -3)$, $B(6, 1)$.

Find a formula that expresses the fact that $P(x, y)$ is a distance 5 from the origin. Discribe the set of all such points.

Find all points on the x-axis that are a distance 5 from the point (-2,4)?

Find the point with coordinates of the form (2a, a) that is in the third quadrant and is a distance 5 from P(1, 3)?

Find all the points with coordinates of the form (a, a) that are a distance 3 from P(-2, 1)?

For what values of a is the distance between P(a, 3) and Q(5, 2a) greater than then $\sqrt{26}$?

Find an equation of the circle that is concentric with $x^2+y^2+4x - 6y + 4 = 0$ and passes through P(2,6).

Determine whether the point P is inside, outside, or on the circle with center C and radius r
P(2, 3), C(4,6), r = 4

For the given circle, find (a) the x-intercepts and (b) y-intercepts.
$x^2 + y^2 - 4x - 6y + 4 = 0$

RADIO BROADCASTING RANGES The signal from a radio station has a circular range of 50 miles. A second radio station, located 100 miles east and 80 miles north of the first station, has a range of 80 miles. Are there locations where signals can be received from both radio stations? Explain your answer.

A circle C_1 of radius 5 has its center at the origin. Outside this circle is a first quadrant circle C_2 of radius 2 that is tangent to C_1. The y-coordinate of the center of C_2 is 3. Find the x-coordinate of the center of C_2.

Find the slope-intercept form of the line that satisfies the given conditions
x-intercept 4, y-intercept –3

Find a general form of an equation for the perpendicular bisector of the segment AB
A (3, –1), B (–6, 4)?

Chapter 3

Constructing a crate The frame for a shipping crate is to be constructed from 24 feet of 2 ×2 lumber (see the figure).
(a) If the crate is to have square ends of side *x* feet, express the outer volume *V* of the crate as a function of *x* (disregard the thickness of the lumber).
(b) Sketch the graph of V for $x > 0$.

Deflections of diving boards A diver stands at the very end of a diving board before beginning a dive. The deflecton d of the board at a position s feet from the stationary end is given by $d=cs^2(3L - s)$ for $0 \le s \le L$, where L is the length of the board and c is a positive constant that depends on the weight of the diver and on the physica properties of the board. Suppose the board is 10 ft long.
a) If the deflection at the end of the board is 1 ft, find c
b) Show that the deflection is .5 foot somewhere between s = 6.5 and s = 6.6

Deer population A herd of 100 deer is introduced onto a small island. At first the herd increases rapidly, but eventually food resources dwindle and the population declines. Suppose that the number N(t) of deer after t years is given by $N(t) = -t^4 + 21t^2 + 100$, where $t > 0$.
a) Determine the values of t for which $N(t) > 0$, and sketch the graph of N.
b) Does the population become extinct? If so, when?

Use synthetic division to find the quotient and remainder if the first polynomial is divided by the second
$2x^3 - 3x^2 + 4x - 5$; $x - 2$

Parabolic arch An arch has the shape of the parabola $y = 4 - x^2$. A rectangle is fit under the arch by selecting a point (x, y) on the parabola.
(a) Express the area *A* of the rectangle in terms of *x*.
(b) If $x = 1$, the rectangle has base 2 and height 3. Find the base of a second rectangle that has the same area.

Dimensions of a capsule An aspirin tablet in the shape of a right circular cylinder has a height of 1/3 cm and a radius of 1/2 cm. The manufacturer also wishes to market the aspirin in capsule form. The capsule is to be 3/2 cm long, in the shape of a right cylinder with hemispheres attached at both ends.

(a) If r denotes the radius of a hemisphere, find a formula for the volume of the capsule.
(b) Find the radius of the capsule so that its volume is equal to that of the tablet.

Find a polynomial f(x) of degree 3 that has the indicated zeros and satisfies the given condition.
–1, 2, 3; f(–2) = 80

Find a polynomial f(x) of degree 4 with leading coefficient 1 such that both –4 and 3 are zeros of multiplicity 2, and sketch the graph of f.

Find the zeros of $f(x)$ and state the multiplicity of each zero.
$f(x) = x^2(3x + 2)(2x - 5)^3$

A storage tank for propane gas is to be constructed in the shape of a right cylinder of altitude 10 feet with a hemisphere attached to each end. Determine the radius x so that the resulting volume is $27\pi \text{ft}^3$.

A storage shelter is to be constructed in the shape of a cube with a triangular prism forming the roof. The length x of a side of the cube is yet to be determined.

a) If the total height of the structure is 6 feet, show that its volume V is given by $V = x^3 + \frac{1}{2}x^2(6 - x)$

b) Determine x so that the volume is 80 ft^3.

Designing a tent A canvas camping tent is to be constructed in the shape of a pyramid with a square base. An 8-foot pole will form the center support, as illustrated in the figure. Find the length x of the base so that the total amount of canvas needed for the sides and bottom is 384 ft^2.