A grapefruit is tossed straight up with an initial velocity of 50 ft/sec. The grapefruit is 5 feet above the ground when it is released. Its height at time t is given by $y = -16t^2 + 50t + 5$. How high does it go before returning to the ground?

When you cough, your windpipe contracts. The speed, v, with which air comes out depends on the radius, r, of your windpipe. If R is the normal (rest) radius of your windpipe, then for $r \le R$, the speed is given by: $v = a(R - r)r^2$
where a is a positive constant.
What value of r maximizes the speed?

For some positive constant C, a patient's temperature change, T , due to a dose, D, of a drug is given by $T = (C/2 - D/3)D^2$.
(a) What dosage maximizes the temperature change?
(b) The sensitivity of the body to the drug is defined as dT/dD. What dosage maximizes sensitivity?

A warehouse selling cement has to decide how often and in what quantities to reorder. It is cheaper, on average, to place large orders, because this reduces the ordering cost per unit. On the other hand, larger orders mean higher storage costs. The warehouse always reorders cement in the same quantity, q. The total weekly cost, C, of ordering and storage is given by
$C = a/q + bq$,
where a, b are positive constants.
(a) Which of the terms, a/q and bq, represents the ordering cost and which represents the storage cost?
(b) What value of q gives the minimum total cost?

A chemical reaction converts substance A to substance Y; the presence of Y catalyzes the reaction. At the start of the reaction, the quantity of A present is a grams. At time t seconds later, the quantity of Y present is y grams. The rate of the reaction, in grams/sec, is given by
Rate $= ky(a - y)$,
k is a positive constant.
(a) For what values of y is the rate nonnegative? Graph the rate against y.
(b) For what values of y is the rate a maximum?

When an electric current passes through two resistors with resistance r 1 and r 2 , connected in parallel, the combined resistance, R, can be calculated from the equation
$1/R = 1/r_1 + 1/r_2$,
where R, r_1, and r_2 are positive. Assume that r_2 is constant.
(a) Show that R is an increasing function of r_1.
(b) Where on the interval $a \le r_1 \le b$ does R take its maximum value?

The distance, s, traveled by a cyclist, who starts at 1 pm, is given in Figure 4.40. Time, t, is in hours since noon.
(a) Explain why the quantity, s/t, is represented by the slope of a line from the origin to the point (t, s) on the graph.
(b) Estimate the time at which the quantity s/t is a maximum.
(c) What is the relationship between the quantity s/t and the instantaneous speed of the cyclist at the time you found in part (b)?

Total cost and revenue are approximated by the functions $C = 5000 + 2.4q$ and $R = 4q$, both in dollars. Identify the fixed cost, marginal cost per item, and the price at which this commodity is sold.

(a) Fixed costs are \$3 million; variable costs are \$0.4 million per item. Write a formula for total cost as a function of quantity, q.

(b) The item in part (a) is sold for \$0.5 million each. Write a formula for revenue as a function of q.
(c) Write a formula for the profit function for this item.

The revenue from selling q items is $R(q) = 500q - q^2$ and the total cost is $C(q) = 150 + 10q$. Write a function that gives the total profit earned, and find the quantity which maximizes the profit.

Revenue is given by $R(q) = 450q$ and cost is given by $C(q) = 10{,}000 + 3q^2$. At what quantity is profit maximized? What is the total profit at this production level?

When production is 2000, marginal revenue is \$4 per unit and marginal cost is \$3.25 per unit. Do you expect maximum profit to occur at a production level above or below 2000? Explain.

The total cost C(q) of producing q goods is given by:
$C(q) = 0.01q^3 - 0.6q^2 + 13q$
(a) What is the fixed cost?
(b) What is the maximum profit if each item is sold for \$7? (Assume you sell everything you produce.)
(c) Suppose exactly 34 goods are produced. They all sell when the price is \$7 each, but for each \$1 increase in price, 2 fewer goods are sold. Should the price be raised, and if so by how much?

(a) A cruise line offers a trip for \$1000 per passenger. If at least 100 passengers sign up, the price is reduced for all the passengers by \$5 for every additional passenger (beyond 100) who goes on the trip. The boat can accommodate 250 passengers. What number of passengers maximizes the cruise line's total revenue? What price does each passenger pay then?
(b) The cost to the cruise line for q passengers is 40,000 + 200q. What is the maximum profit that the cruise line can make on one trip? How many passengers must sign up for the maximum to be reached and what price will each pay?

Figure 4.69 shows the curves $y = \sqrt{x}$, $x = 9$, $y = 0$, and a rectangle with its sides parallel to the axes and its left end at $x = a$. Find the dimensions of the rectangle having the maximum possible area.

The hypotenuse of a right triangle has one end at the origin and one end on the curve $y = x^2e^{-3x}$, with $x \geq 0$. One of the other two sides is on the x-axis, the other side is parallel to the y-axis. Find the maximum area of such a triangle. At what x-value does it occur?

A rectangle has one side on the x-axis and two vertices on the curve $y = 1/(1 + x^2)$. Find the vertices of the rectangle with maximum area.

A rectangle has one side on the x-axis, one side on the y-axis, one vertex at the origin and one on the curve $y = e^{-2x}$ for $x \geq 0$. Find the
(a) Maximum area
(b) Minimum perimeter

A hemisphere of radius 1 sits on a horizontal plane. A cylinder stands with its axis vertical, the center of its base at the center of the sphere, and its top circular rim touching the hemisphere. Find the radius and height of the cylinder of maximum volume.

A closed box has a fixed surface area A and a square base with side x.
(a) Find a formula for its volume, V, as a function of x.
(b) Sketch a graph of V against x.
(c) Find the maximum value of V.

If you have 100 feet of fencing and want to enclose a rectangular area up against a long, straight wall, what is the largest area you can enclose?

A rectangular beam is cut from a cylindrical log of radius 30 cm. The strength of a beam of width w and height h is proportional to wh^2 . (See Figure 4.70.) Find the width and height of the beam of maximum strength.

A landscape architect plans to enclose a 3000 square foot rectangular region in a botanical garden. She will use shrubs costing \$25 per foot along three sides and fencing costing \$10 per foot along the fourth side. Find the minimum total cost.

A rectangular swimming pool is to be built with an area of 1800 square feet. The owner wants 5-foot wide decks along either side and 10-foot wide decks at the two ends. Find the dimensions of the smallest piece of property on which the pool can be built satisfying these conditions.

A square-bottomed box with no top has a fixed volume, V. What dimensions minimize the surface area?

A light is suspended at a height h above the floor. (See Figure 4.71.) The illumination at the point P is inversely proportional to the square of the distance from the point P to the light and directly proportional to the cosine of the angle θ. How far from the floor should the light be to maximize the illumination at the point P?

Which point on the parabola $y = x^2$ is nearest to (1, 0)? Find the coordinates to two decimals. [Hint: Minimize the square of the distance—this avoids square roots.]

Find the coordinates of the point on the parabola $y = x^2$ which is closest to the point (3,0).

The cross-section of a tunnel is a rectangle of height h surmounted by a semicircular roof section of radius r (See Figure 4.72 on page 204). If the cross-sectional area is A, determine the dimensions of the cross section which minimize the perimeter.

Of all rectangles with given area, A, which has the shortest diagonals?

You run a small furniture business. You sign a deal with a customer to deliver up to 400 chairs, the exact number to be determined by the customer later. The price will be \$90 per chair up to 300 chairs, and above 300, the price will be reduced by \$0.25 per chair (on the whole order) for every additional chair over 300 ordered. What are the largest and smallest revenues your company can make under this deal?

The cost of fuel to propel a boat through the water (in dollars per hour) is proportional to the cube of the speed. A certain ferry boat uses \$100 worth of fuel per hour when cruising at 10 miles per hour. Apart from fuel, the cost of running this ferry (labor, maintenance, and so on) is \$675 per hour. At what speed should it travel so as to minimize the cost per mile traveled?

On the same side of a straight river are two towns, and the townspeople want to build a pumping station, S. See Figure 4.74. The pumping station is to be at the river's edge with pipes extending straight to the two towns. Where should the pumping station be located to minimize the total length of pipe?

A pigeon is released from a boat (point B in Figure 4.75) floating on a lake. Because of falling air over the cool water, the energy required to fly one meter over the lake is twice the corresponding energy e

required for flying over the bank (e = 3 joule/meter). To minimize the energy required to fly from B to the loft, L, the pigeon heads to a point P on the bank and then flies along the bank to L. The distance AL is 2000 m, and AB is 500 m.
(a) Express the energy required to fly from B to L via P as a function of the angle θ (the angle BP A).
(b) What is the optimal angle θ?
(c) Does your answer change if AL, AB, and e have different numerical values?

To get the best view of the Statue of Liberty in Figure 4.76, you should be at the position where θ is a maximum. If the statue stands 92 meters high, including the pedestal, which is 46 meters high, how far from the base should you be? [Hint: Find a formula for θ in terms of your distance from the base. Use this function to maximize θ, noting that $0 \le \theta \le \pi/2$.]

A light ray starts at the origin and is reflected off a mirror along the line y = 1 to the point (2, 0). See Figure 4.77. Fermat's Principle 6 says that light's path minimizes the time of travel. The speed of light is a constant.
(a) Using Fermat's principle, find the optimal position of P .
(b) Using your answer to part (a), derive the Law of Reflection, that $\theta_1 = \theta_2$.

Minimize $x^2 + y^2$ while satisfying x + y = 4 using the following steps.
(a) Graph x+y = 4. On the same axes, graph $x^2 + y^2 = 1$, $x^2 + y^2 = 4$, $x^2 + y^2 = 9$.
(b) Explain why the minimum value of $x^2 + y^2$ on x + y = 4 occurs at the point at which a graph of $x^2 + y^2$ = Constant is tangent to the line x + y = 4.
(c) Using your answer to part (b) and implicit differentiation to find the slope of the circle, find the minimum value of $x^2 + y^2$ such that x + y = 4.

The quantity Q of an item which can be produced from quantities x and y of two raw materials is given by Q = 10xy at a cost of C = x + 2y thousand dollars. If there is a budget of $10 thousand for raw materials, find the maximum production using the following steps.
(a) Graph x+2y = 10 in the first quadrant. On the same axes, graph Q = 10xy = 100, Q = 10xy = 200, and Q = 10xy = 300.
(b) Explain why the maximum production occurs at a point at which a production curve is tangent to the cost line C = 10.
(c) Using your answer to part (b) and implicit differentiation to find the slope of the curve, find the maximum production under this budget.

According to the US Census, the world population P , in billions, is approximately $P = 6.342e^{0.011t}$, where t is in years since January 1, 2004. At what rate was the world's population increasing on that date? Give your answer in millions of people per year.

With time, t, in minutes, the temperature, H, in degrees Celsius, of a bottle of water put in the refrigerator at t = 0 is given by $H = 4 + 16e^{-0.02t}$. How fast is the water cooling initially? After 10 minutes? Give units.

The power, P, dissipated when a 9-volt battery is put across a resistance of R ohms is given by P = 81/R. What is the rate of change of power with respect to resistance?

With length, l, in meters, the period T, in seconds, of a pendulum is given by
$T = 2\pi\sqrt{l/9.8}$.
(a) How fast does the period increase as *l* increases?
(b) Does this rate of change increase or decrease as *l* increases?

At time t, in hours, a lake is covered with ice of thickness y cm, where $y = 0.2t^{1.5}$.

(a) How fast is the ice forming when $t = 1$? When $t = 2$? Give units.
(b) If ice forms for $0 \leq t \leq 3$, at what time in this interval is the ice thickest? At what time is the ice forming fastest?

A dose, D, of a drug causes a temperature change, T, in a patient. For C a positive constant, T is given by $T = (C/2 - D/3)D^3$.
(a) What is the rate of change of temperature change with respect to dose?
(b) For what doses does the temperature change increase as the dose increases?

The average cost per item, C, in dollars, of manufacturing a quantity q of cell phones is given by
$C = a/q + b$
where a, b are positive constants.
(a) Find the rate of change of C as q increases. What are its units?
(b) If production increases at a rate of 100 cell phones per week, how fast is the average cost changing? Is the average cost increasing or decreasing?

For positive constants A and B, the force, F , between two atoms in a molecule at a distance r apart is given by $F = -A/r^2 + B/r^3$.
(a) How fast does force change as r increases? What type of units does it have?
(b) If at some time t the distance is changing at a rate k, at what rate is the force changing with time? What type of units does this rate of change have?

An item costs \$500 at time t = 0 and costs \$P in year t. When inflation is r% per year, the price is given by P = 500e^(rt/100).
(a) If r is a constant, at what rate is the price rising (in dollars per year)
(i) Initially? (ii) After 2 years?
(b) Now suppose that r is increasing by 0.3 per year when r = 4 and t = 2. At what rate (dollars per year) is the price increasing at that time?

A voltage V across a resistance R generates a current I = V/R. If a constant voltage of 9 volts is put across a resistance that is increasing at a rate of 0.2 ohms per second when the resistance is 5 ohms, at what rate is the current changing?

Coroners estimate time of death using the rule of thumb that a body cools about 2◦F during the first hour after death and about 1◦F for each additional hour. Assuming an air temperature of 68◦F and a living body temperature of 98.6◦F , the temperature T (t) in ◦F of a body at a time t hours since death is given by $T(t) = 68 + 30.6e^{-kt}$.
(a) For what value of k will the body cool by 2◦F in the first hour?
(b) Using the value of k found in part (a), after how many hours will the temperature of the body be decreasing at a rate of 1◦F per hour?
(c) Using the value of k found in part (a), show that, 24 hours after death, the coroner's rule of thumb gives approximately the same temperature as the formula.

A certain quantity of gas occupies a volume of 20 cm^3 at a pressure of 1 atmosphere. The gas expands without the addition of heat, so, for some constant k, its pressure, P , and volume, V , satisfy the relation $PV^{1.4} = k$.
(a) Find the rate of change of pressure with volume. Give units.
(b) The volume is increasing at 2 cm^3/min when the volume is 30 cm^3 . At that moment, is the pressure increasing or decreasing? How fast? Give units.

(a) A hemispherical bowl of radius 10 cm contains water to a depth of h cm. Find the radius of the surface of the water as a function of h.
(b) The water level drops at a rate of 0.1 cm per hour. At what rate is the radius of the water decreasing when the depth is 5 cm?

A cone-shaped coffee filter of radius 6 cm and depth 10 cm contains water, which drips out through a hole at the bottom at a constant rate of 1.5 cm^3 per second.
(a) If the filter starts out full, how long does it take to empty?
(b) Find the volume of water in the filter when the depth of the water is h cm.
(c) How fast is the water level falling when the depth is 8 cm?

A spherical snowball is melting. Its radius is decreasing at 0.2 cm per hour when the radius is 15 cm. How fast is its volume decreasing at that time?

A ruptured oil tanker causes a circular oil slick on the surface of the ocean. When its radius is 150 meters, the radius of the slick is expanding by 0.1 meter/minute and its thickness is 0.02 meter. At that moment:
(a) How fast is the area of the slick expanding?
(b) The circular slick has the same thickness everywhere, and the volume of oil spilled remains fixed. How fast is the thickness of the slick decreasing?

A potter forms a piece of clay into a cylinder. As he rolls it, the length, L, of the cylinder increases and the radius, r, decreases. If the length of the cylinder is increasing at 0.1 cm per second, find the rate at which the radius is changing when the radius is 1 cm and the length is 5 cm.

The London Eye is a large ferris wheel that has diameter 135 meters and revolves continuously. Passengers enter the cabins at the bottom of the wheel and complete one revolution in 20 minutes. One minute into the ride a passenger is rising at 0.1 meters per second. How fast is the horizontal motion of the passenger at that moment?

A gas station stands at the intersection of a north-south road and an east-west road. A police car is traveling toward the gas station from the east, chasing a stolen truck which is traveling north away from the gas station. The speed of the police car is 100 mph at the moment it is 3 miles from the gas station. At the same time, the truck is 4 miles from the gas station going 80 mph. At this moment:
(a) Is the distance between the car and truck increasing or decreasing? How fast? (Distance is measured along a straight line joining the car and the truck.)
(b) How does your answer change if the truck is going 70 mph instead of 80 mph?

A train is traveling at 0.8 km/min along a long straight track, moving in the direction shown in Figure 4.86. A movie camera, 0.5 km away from the track, is focused on the train.
(a) Express z, the distance between the camera and the train, as a function of x.
(b) How fast is the distance from the camera to the train changing when the train is 1 km from the camera? Give units.
(c) How fast is the camera rotating (in radians/min) at the moment when the train is 1 km from the camera?

A lighthouse is 2 km from the long, straight coastline shown in Figure 4.87. Find the rate of change of the distance of the spot of light from the point O with respect to the angle θ.

A train is heading due west from St. Louis. At noon, a plane flying horizontally due north at a fixed altitude of 4 miles passes directly over the train. When the train has traveled another mile, it is going

80 mph, and the plane has traveled another 5 miles and is going 500 mph. At that moment, how fast is the distance between the train and the plane increasing?

The radius of a spherical balloon is increasing by 2 cm/sec. At what rate is air being blown into the balloon at the moment when the radius is 10 cm? Give units in your answer.

A spherical cell is growing at a constant rate of 400 μm 3 /day (1 μm= 10^{-6} m). At what rate is its radius increasing when the radius is 10 μm?

A raindrop is a perfect sphere with radius r cm and surface area S cm^2 . Condensation accumulates on the raindrop at a rate equal to kS, where k = 2 cm/sec. Show that the radius of the raindrop increases at a constant rate and find that rate.

Sand falls from a hopper at a rate of 0.1 cubic meters per hour and forms a conical pile beneath. If the side of the cone makes an angle of $\pi/6$ radians with the vertical, find the rate at which the height of the cone increases. At what rate does the radius of the base increase? Give both answers in terms of h, the height of the pile in meters.

A circular region is irrigated by a 20 meter long pipe, fixed at one end and rotating horizontally, spraying water. One rotation takes 5 minutes. A road passes 30 meters from the edge of the circular area. See Figure 4.88.

A circular region is irrigated by a 20 meter long pipe, fixed at one end and rotating horizontally, spraying water. One rotation takes 5 minutes. A road passes 30 meters from the edge of the circular area. See Figure 4.88.
(a) How fast is the end of the pipe, P , moving?
(b) How fast is the distance P Q changing when θ is $\pi/2$? When θ is 0?

A water tank is in the shape of an inverted cone with depth 10 meters and top radius 8 meters. Water is flowing into the tank at 0.1 cubic meters/min but leaking out at a rate of $0.001h^2$ cubic meters/min, where h is the depth of the water in the tank in meters. Can the tank ever overflow?

For the amusement of the guests, some hotels have elevators on the outside of the building. One such hotel is 300 feet high. You are standing by a window 100 feet above the ground and 150 feet away from the hotel, and the elevator descends at a constant speed of 30 ft/sec, starting at time t = 0, where t is time in seconds. Let θ be the angle between the line of your horizon and your line of sight to the elevator. (See Figure 4.89.)
(a) Find a formula for h(t), the elevator's height above the ground as it descends from the top of the hotel.
(b) Using your answer to part (a), express θ as a function of time t and find the rate of change of θ with respect to t.
(c) The rate of change of θ is a measure of how fast the elevator appears to you to be moving. At what height is the elevator when it appears to be moving fastest?

In a romantic relationship between Angela and Brian, who are unsuited for each other, a(t) represents the affection Angela has for Brian at time t days after they meet, while b(t) represents the affection Brian has for Angela at time t. If $a(t) > 0$ then Angela likes Brian; if $a(t) < 0$ then Angela dislikes Brian; if $a(t) = 0$ then Angela neither likes nor dislikes Brian. Their affection for each other is given by the relation $a^2(t) + b^2(t) = c$,
where c is a constant.
(a) Show that $a(t) \cdot a'(t) = -b(t) \cdot b'(t)$.

(b) At any time during their relationship, the rate per day at which Brian's affection for Angela changes is $b'(t) = -a(t)$. Explain what this means if Angela
(i) Likes Brian, (ii) Dislikes Brian.
(c) Use parts (a) and (b) to show that $a'(t) = b(t)$. Explain what this means if Brian
(i) Likes Angela, (ii) Dislikes Angela.
(d) If $a(0) = 1$ and $b(0) = 1$ who first dislikes the other?

A drug is injected into a patient at a rate given by $r(t) = ate^{-bt}$ ml/sec, where t is in seconds since the injection started, $0 \le t \le 5$, and a and b are constants. The maximum rate of 0.3 ml/sec occurs half a second after the injection starts. Find a formula for a and b.

Any body radiates energy at various wavelengths. Figure 4.110 shows the intensity of the radiation of a black body at a temperature T = 3000◦ kelvin as a function of the wavelength. The intensity of the radiation is highest in the infrared range, that is, at wavelengths longer than that of visible light (0.4–0.7μm). Max Planck's radiation law, announced to the Berlin Physical Society on October 19, 1900, states that

$$r(\lambda) = \frac{a}{\lambda^5 (e^{b/\lambda} - 1)}.$$

Find constants a and b so that the formula fits the graph. (Later in 1900 Planck showed from theory that $a = 2\pi c^2 h$ and $b = hc/(Tk)$ where c = speed of light, h = Planck's constant, and k = Boltzmann's constant.)

A right triangle has one vertex at the origin and one vertex on the curve $y = e^{-x/3}$ for $1 \le x \le 5$. One of the two perpendicular sides is along the x-axis; the other is parallel to the y-axis. Find the maximum and minimum areas for such a triangle.

A rectangle has one side on the x-axis and two corners on the top half of the circle of radius 1 centered at the origin. Find the maximum area of such a rectangle. What are the coordinates of its vertices?

A square-bottomed box with a top has a fixed volume, V. What dimensions minimize the surface area?

A business sells an item at a constant rate of r units per month. It reorders in batches of q units, at a cost of a+bq dollars per order. Storage costs are k dollars per item per month, and, on average, q/2 items are in storage, waiting to be sold. [Assume r, a, b, k are positive constants.]
(a) How often does the business reorder?
(b) What is the average monthly cost of reordering?
(c) What is the total monthly cost, C of ordering and storage?
(d) Obtain Wilson's lot size formula, the optimal batch size which minimizes cost.

A ship is steaming due north at 12 knots (1 knot = 1.85 kilometers/hour) and sights a large tanker 3 kilometers away northwest steaming at 15 knots due east. For reasons of safety, the ships want to maintain a distance of at least 100 meters between them. Use a calculator or computer to determine the shortest distance between them if they remain on their current headings, and hence decide if they need to change course.

Boise, Idaho, is about 300 miles inland from the nearest point on the Pacific coast; San Diego is about 1000 miles south of that point down the coast. (See Figure 4.111.) Assuming the coast is a straight line going north-south, C is the point along the coast directly west of Boise. It costs 2 cents per mile to transport a ton of potatoes by truck and 1 cent per mile to transport them by ship. The Idaho Potato Company wants to find the point, P , on the Pacific coast to which it should truck its potatoes before loading them aboard a cargo ship in order to minimize the total cost of transporting them from Boise to San Diego.

(a) Set up the function which The Idaho Potato Company must minimize.
(b) Find the position of P which minimizes cost.

A polystyrene cup is in the shape of a frustum (the part of a cone between two parallel planes cutting the cone), has top radius 2r, base radius r and height h. The surface area S of such a cup is given by S = $3\pi r\sqrt{r^2+h^2}$ and its volume V by V = $7\pi r^2h/3$. If the cup is to hold 200 ml, use a calculator or a computer to estimate the value of r that minimizes its surface area.

For a > 0, the following line forms a triangle in the first quadrant with the x- and y-axes:
$a(a^2 + 1)y = a - x$.
(a) In terms of a, find the x- and y-intercepts of the line.
(b) Find the area of the triangle, as a function of a.
(c) Find the value of a making the area a maximum.
(d) What is this greatest area?
(e) If you want the triangle to have area 1/5, what choices do you have for a?

A piece of wire of length L cm is cut into two pieces. One piece, of length x cm, is made into a circle; the rest is made into a square.
(a) Find the value of x that makes the sum of the areas of the circle and square a minimum. Find the value of x giving a maximum.
(b) For the values of x found in part (a), show that the ratio of the length of wire in the square to the length of wire in the circle is equal to the ratio of the area of the square to the area of the circle. 9
(c) Are the values of x found in part (a) the only values of x for which the ratios in part (b) are equal?

A spherical balloon is inflated so that its radius is increasing at a constant rate of 1 cm per second. At what rate is air being blown into the balloon when its radius is 5 cm?

When the growth of a spherical cell depends on the flow of nutrients through the surface, it is reasonable to assume that the growth rate, dV/dt, is proportional to the surface area, S. Assume that for a particular cell $dV/dt = 1/3 \cdot S$. At what rate is its radius r increasing?

A horizontal disk of radius a centered at the origin in the xy-plane is rotating about a vertical axis through the center. The angle between the positive x-axis and a radial line painted on the disk is θ radians.
(a) What does $d\theta/dt$ represent?
(b) What is the relationship between $d\theta/dt$ and the speed v of a point on the rim?

A chemical storage tank is in the shape of an inverted cone with depth 12 meters and top radius 5 meters. When the depth of the chemical in the tank is 1 meter, the level is falling at 0.1 meters per minute. How fast is the volume of chemical changing?

A voltage, V volts, applied to a resistor of R ohms produces an electric current of I amps where V=IR. As the current flows the resistor heats up and its resistance falls. If 100 volts is applied to a resistor of 1000 ohms the current is initially 0.1 amps but rises by 0.001 amps/minute. At what rate is the resistance falling if the voltage remains constant?

A radio navigation system used by aircraft gives a cockpit readout of the distance, s, in miles, between a fixed ground station and the aircraft. The system also gives a readout of the instantaneous rate of change, ds/dt, of this distance in miles/hour. An aircraft on a straight flight path at a constant altitude of 10,560 feet (2 miles) has passed directly over the ground station and is now flying away from it. What is the speed of this aircraft along its constant altitude flight path when the cockpit readouts are s = 4.6 miles and ds/dt = 210 miles/hour?

A fixed quantity of gas is allowed to expand at constant temperature. Find the rate of change of pressure with respect to volume.

A certain quantity of gas occupies 10 cm^3 at a pressure of 2 atmospheres. The pressure is increased, while keeping the temperature constant.
(a) Does the volume increase or decrease?
(b) If the pressure is increasing at a rate of 0.05 atmospheres/minute when the pressure is 2 atmospheres, find the rate at which the volume is changing at that moment. What are the units of your answer?

A student is speeding down Route 11 in his fancy red Porsche when his radar system warns him of an obstacle 400 feet ahead. He immediately applies the brakes, starts to slow down, and spots a skunk in the road directly ahead of him. The "black box" in the Porsche records the car's speed every two seconds, producing the following table. The speed decreases throughout the 10 seconds it takes to stop, although not necessarily at a uniform rate.
(a) What is your best estimate of the total distance the student's car traveled before coming to rest?
(b) Which one of the following statements can you justify from the information given?
(i) The car stopped before getting to the skunk.
(ii) The "black box" data is inconclusive. The skunk may or may not have been hit.
(iii) The skunk was hit by the car.

Roger runs a marathon. His friend Jeff rides behind him on a bicycle and clocks his speed every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. Jeff's data follow:
(a) Assuming that Roger's speed is never increasing, give upper and lower estimates for the distance Roger ran during the first half hour.
(b) Give upper and lower estimates for the distance Roger ran in total during the entire hour and a half.
(c) How often would Jeff have needed to measure Roger's speed in order to find lower and upper estimates within 0.1 mile of the actual distance he ran?

A car initially going 50 ft/sec brakes at a constant rate (constant negative acceleration), coming to a stop in 5 seconds.
(a) Graph the velocity from $t = 0$ to $t = 5$.
(b) How far does the car travel?
(c) How far does the car travel if its initial velocity is doubled, but it brakes at the same constant rate?

A woman drives 10 miles, accelerating uniformly from rest to 60 mph. Graph her velocity versus time. How long does it take for her to reach 60 mph?

Two cars start at the same time and travel in the same direction along a straight road. Figure 5.13 gives the velocity, v, of each car as a function of time, t. Which car:
(a) Attains the larger maximum velocity?
(b) Stops first?
(c) Travels farther?

Two cars travel in the same direction along a straight road. Figure 5.14 shows the velocity, v, of each car at time t. Car B starts 2 hours after car A and car B reaches a maximum velocity of 50 km/hr.
(a) For approximately how long does each car travel?
(b) Estimate car A's maximum velocity.
(c) Approximately how far does each car travel?

Oil leaks out of a tanker at a rate of r = f(t) gallons per minute, where t is in minutes. Write a definite integral expressing the total quantity of oil which leaks out of the tanker in the first hour.

Coal gas is produced at a gasworks. Pollutants in the gas are removed by scrubbers, which become less and less efficient as time goes on. The following measurements, made at the start of each month, show the rate at which pollutants are escaping (in tons/month) in the gas:
(a) Make an overestimate and an underestimate of the total quantity of pollutants that escape during the first month.
(b) Make an overestimate and an underestimate of the total quantity of pollutants that escape during the six months.
(c) How often would measurements have to be made to find overestimates and underestimates which differ by less than 1 ton from the exact quantity of pollutants that escaped during the first six months?

A two-day environmental clean up started at 9 am on the first day. The number of workers fluctuated as shown in Figure 5.36. If the workers were paid \$10 per hour, how much was the total personnel cost of the clean up?

Suppose in Problem 18 that the workers were paid \$10 per hour for work during the time period 9 am to 5 pm and were paid \$15 per hour for work during the rest of the day. What would the total personnel costs of the clean up have been under these conditions?

A warehouse charges its customers \$5 per day for every 10 cubic feet of space used for storage. Figure 5.37 records the storage used by one company over a month. How much will the company have to pay?

A bicyclist pedals along a straight road with velocity, v, given in Figure 5.45. She starts 5 miles from a lake; positive velocities take her away from the lake and negative velocities take her toward the lake. When is the cyclist farthest from the lake, and how far away is she then?

A car speeds up at a constant rate from 10 to 70 mph over a period of half an hour. Its fuel efficiency (in miles per gallon) at various speeds is shown in the table. Make lower and upper estimates of the quantity of fuel used during the half hour.

Height velocity graphs are used by endocrinologists to follow the progress of children with growth deficiencies. Figure 5.46 shows the height velocity curves of an average boy and an average girl between age 3 and 18.
(a) Which curve is for girls and which is for boys? Explain how you can tell.
(b) About how much does the average boy grow between ages 3 and 10?
(c) The growth spurt associated with adolescence and the onset of puberty occurs between ages 12 and 15 for the average boy and between ages 10 and 12.5 for the average girl. Estimate the height gained by each average child during this growth spurt.
(d) When fully grown, about how much taller is the average man than the average woman? (The average boy and girl are about the same height at age 3.)

A village wishes to measure the quantity of water that is piped to a factory during a typical morning. A gauge on the water line gives the flowrate (in cubic meters per hour) at any instant. The flowrate is about 100 m^3/hr at 6 am and increases steadily to about 280 m^3/hr at 9 am.
(a) Using only this information, give your best estimate of the total volume of water used by the factory between 6 am and 9 am.
(b) How often should the flowrate gauge be read to obtain an estimate of this volume to within 6 m^3?

You jump out of an airplane. Before your parachute opens you fall faster and faster, but your acceleration decreases as you fall because of air resistance. The table gives your acceleration, a (in m/sec^2), after t seconds.
(a) Give upper and lower estimates of your speed at t = 5.
(b) Get a new estimate by taking the average of your upper and lower estimates. What does the concavity of the graph of acceleration tell you about your new estimate?

A car going 80 ft/sec (about 55 mph) brakes to a stop in 8 seconds. Its velocity is recorded every 2 seconds and is given in the following table.
(a) Give your best estimate of the distance traveled by the car during the 8 seconds.
(b) To estimate the distance traveled accurate to within 20 feet, how often should you record the velocity?

Annual coal production in the US (in quadrillion BTU per year) is given in the table. 5 Estimate the total amount of coal produced in the US between 1960 and 1990. If r = f(t) is the rate of coal production t years since 1960, write an integral to represent the 1960–1990 coal production.

An old rowboat has sprung a leak. Water is flowing into the boat at a rate, r(t), given in the following table.
(a) Compute upper and lower estimates for the volume of water that has flowed into the boat during the 15 minutes.
(b) Draw a graph to illustrate the lower estimate.

Figure 5.73 gives your velocity during a trip starting from home. Positive velocities take you away from home and negative velocities take you toward home. Where are you at the end of the 5 hours? When are you farthest from home? How far away are you at that time?

A bicyclist is pedaling along a straight road for one hour with a velocity v shown in Figure 5.74. She starts out five kilometers from the lake and positive velocities take her toward the lake. [Note: The vertical lines on the graph are at 10 minute (1/6 hour) intervals.]

(a) Does the cyclist ever turn around? If so, at what time(s)?
(b) When is she going the fastest? How fast is she going then? Toward the lake or away?
(c) When is she closest to the lake? Approximately how close to the lake does she get?
(d) When is she farthest from the lake? Approximately how far from the lake is she then?

Figure 5.75 shows the rate of change of the quantity of water in a water tower, in liters per day, during the month of April. If the tower had 12,000 liters of water in it on April 1, estimate the quantity of water in the tower on April 30.

A bar of metal is cooling from 1000◦C to room temperature, 20◦C. The temperature, H, of the bar t minutes after it starts cooling is given, in ◦C, by
$H = 20 + 980e^{-0.1t}$.
(a) Find the temperature of the bar at the end of one hour.
(b) Find the average value of the temperature over the first hour.
(c) Is your answer to part (b) greater or smaller than the average of the temperatures at the beginning and the end of the hour? Explain this in terms of the concavity of the graph of H.

A cup of coffee at 90◦C is put into a 20◦C room when t = 0. The coffee's temperature is changing at a rate of $r(t) = -7e^{-0.1t}$ ◦C per minute, with t in minutes. Estimate the coffee's temperature when t = 10.

Water is pumped out of a holding tank at a rate of $5 - 5e^{-0.12t}$ liters/minute, where t is in minutes since the pump is started. If the holding tank contains 1000 liters of water when the pump is started, how much water does it hold one hour later?

Water is run into a large tank through a hose at a constant rate. After 5 minutes a hole is opened in the bottom of the tank, and water starts to flow out. Initially the flow rate through the hole is twice as great as the rate through the hose, but as the water level in the tank goes down, the flow rate through the hole decreases; after another 10 minutes the water level in the tank appears to be constant. Plot graphs of the flow rates through the hose and through the hole against time on the same pair of axes. Show how the volume of water in the tank at any time can be interpreted as an area (or the difference between two areas) on the graph. In particular, interpret the steady-state volume of water in the tank.

Figure 5.80 shows thrust-time curves for two model rockets. The thrust or force, F , of the engine (in newtons) is plotted against time, t, (in seconds). The total impulse of the rocket's engine is defined as the definite integral of F with respect to t. The total impulse is a measure of the strength of the engine.
(a) For approximately how many seconds is the thrust of rocket B greater than 10 newtons?
(b) Estimate the total impulse for model rocket A.
(c) What are the units for the total impulse calculated in part (b)?
(d) Which rocket has the largest total impulse?
(e) Which rocket has the largest maximum thrust?

The Glen Canyon Dam at the top of the Grand Canyon prevents natural flooding. In 1996, scientists decided an artificial flood was necessary to restore the environmental balance. Water was released through the dam at a controlled rate 7 shown in Figure 5.81. The figure also shows the rate of flow of the last natural flood in 1957.
(a) At what rate was water passing through the dam in 1996 before the artificial flood?
(b) At what rate was water passing down the river in the pre-flood season in 1957?
(c) Estimate the maximum rates of discharge for the 1996 and 1957 floods.
(d) Approximately how long did the 1996 flood last? How long did the 1957 flood last?
(e) Estimate how much additional water passed down the river in 1996 as a result of the artificial flood.
(f) Estimate how much additional water passed down the river in 1957 as a result of the flood.

A mouse moves back and forth in a straight tunnel, attracted to bits of cheddar cheese alternately introduced to and removed from the ends (right and left) of the tunnel. The graph of the mouse's velocity, v, is given in Figure 5.83, with positive velocity corresponding to motion toward the right end. Assuming that the mouse starts (t = 0) at the center of the tunnel, use the graph to estimate the time(s) at which:
(a) The mouse changes direction.
(b) The mouse is moving most rapidly to the right; to the left.
(c) The mouse is farthest to the right of center; farthest to the left.
(d) The mouse's speed (i.e., the magnitude of its velocity) is decreasing.
(e) The mouse is at the center of the tunnel.

When an aircraft attempts to climb as rapidly as possible, its climb rate decreases with altitude. (This occurs because the air is less dense at higher altitudes.) The table shows performance data for a single-engine aircraft.
(a) Calculate upper and lower estimates for the time required for this aircraft to climb from sea level to 10,000 ft.
(b) If climb rate data were available in increments of 500 ft, what would be the difference between a lower and upper estimate of climb time based on 20 subdivisions?

Pollution is being dumped into a lake at a rate which is increasing at a constant rate from 10 kg/year to 50 kg/year until a total of 270 kg has been dumped. Sketch a graph of the rate at which pollution is being dumped in the lake against time. How long does it take until 270 kg has been dumped?

In drilling an oil well, the total cost, C, consists of fixed costs (independent of the depth of the well) and marginal costs, which depend on depth; drilling becomes more expensive, per meter, deeper into the earth. Suppose the fixed costs are 1,000,000 riyals (the riyal is the unit of currency of Saudi Arabia), and the marginal costs are
$C'(x) = 4000 + 10x$ riyals/meter,
where x is the depth in meters. Find the total cost of drilling a well x meters deep.

One of the earliest pollution problems brought to the attention of the Environmental Protection Agency (EPA) was the case of the Sioux Lake in eastern South Dakota. For years a small paper plant located nearby had been discharging waste containing carbon tetrachloride (CCl_4) into the waters of the lake. At the time the EPA learned of the situation, the chemical was entering at a rate of 16 cubic yards/year.

The agency ordered the installation of filters designed to slow (and eventually stop) the flow of CCl_4 from the mill. Implementation of this program took exactly three years, during which the flow of pollutant was steady at 16 cubic yards/year. Once the filters were installed, the flow declined. If t is time measured in years since the EPA learned of the situation, between the time the filters were installed and the time the flow stopped, the rate of flow was well approximated by
Rate (in cubic yards/year) = $t^2 - 14t + 49$.
(a) Graph the rate of CCl_4 flow into the lake as a function of time, beginning at the time the EPA first learned of the situation.
(b) How many years elapsed between the time the EPA learned of the situation and the time the pollution flow stopped entirely?
(c) How much CCl_4 entered the waters during the time shown in the graph in part (a)?

A tomato is thrown upward from a bridge 25 m above the ground at 40 m/sec.
(a) Give formulas for the acceleration, velocity, and height of the tomato at time t.
(b) How high does the tomato go, and when does it reach its highest point?
(c) How long is it in the air?

Ice is forming on a pond at a rate given by $dy/dt = k\sqrt{t}$ where y is the thickness of the ice in inches at time t measured in hours since the ice started forming, and k is a positive constant. Find y as a function of t.

A water balloon launched from the roof of a building at $t = 0$ has vertical velocity $v(t) = -32t + 40$ feet/sec at time t seconds, with $v > 0$ corresponding to upward motion.
a) If the roof of the building is 30ft above the ground, find an expression for the height of the water balloon above the ground at time t.
b) What is the average velocity of the balloon between $t = 1.5$ and $t = 3$ seconds?
c) A 6 foot tall person is standing on the ground. How fast is the water balloon falling when it strikes this person on the top of the head?

If a car goes from 0 to 80 mph in six seconds with constant acceleration, what is that acceleration?

A car starts from rest at time t=0 and accelerates at $-0.6t+4$ meters/sec^2 for $0 \le t \le 12$. How long does it take for the car to go 100 meters?

A car going 80 ft/sec (about 55 mph) brakes to a stop in five seconds. Assume the deceleration is constant.
(a) Graph the velocity against time, t, for $0 < t < 5$ seconds.
(b) Represent, as an area on the graph, the total distance traveled from the time the brakes are applied until the car comes to a stop.
(c) Find this area and hence the distance traveled.
(d) Now find the total distance traveled using antidifferentiation.

A 727 jet needs to attain a speed of 200 mph to take off. It it can accelerate from 0 to 200 mph in 30 seconds, how long must the runway be?

A car going at 30 ft/sec decelerates at a constant 5 ft/sec^2.
(a) Draw up a table showing the velocity of the car every half second. When does the car come to rest?
(b) Using your table, find left and right sums which estimate the total distance traveled before the car comes to rest. Which is an overestimate, and which is an underestimate?
(c) Sketch a graph of velocity against time. On the graph, show an area representing the distance traveled before the car comes to rest. Use the graph to calculate this distance.
(d) Now find a formula for the velocity of the car as a function of time, and then find the total distance traveled by antidifferentiation. What is the relationship between your answer to parts (c) and (d) and your estimates in part (b)?

An object is shot vertically upward from the ground with a velocity of 160 ft/sec.
(a) Sketch a graph of the velocity of the object (with upward as positive) against time.
(b) Mark on the graph the points when the object reaches its highest point and when it lands.
(c) Find the maximum height reached by the object by considering an area on the graph.
(d) Now express velocity as a function of time, and find the greatest height by antidifferentiation.

A stone thrown upward from the top of a 320-foot cliff at 128 ft/sec eventually falls to the beach below.
(a) How long does the stone take to reach its highest point?
(b) What is its maximum height?
(c) How long before the stone hits the beach?
(d) What is the velocity of the stone on impact?

On the moon, the acceleration due to gravity is about 1.6 m/sec^2 (compared to $g \approx 9.8$ m/sec^2 on earth). If you drop a rock on the moon (with initial velocity 0), find formulas for:
(a) Its velocity, $v(t)$, at time t.
(b) The distance, $s(t)$, it falls in time t.

(a) Imagine throwing a rock straight up in the air. What is its initial velocity if the rock reaches a maximum height of 100 feet above its starting point?
(b) Now imagine being transplanted to the moon and throwing a moon rock vertically upward with the same velocity as in part (a). How high will it go? (On the moon, $g = 5$ ft/sec^2.)

A cat, walking along the window ledge of a New York apartment, knocks off a flower pot, which falls to the street 200 feet below. How fast is the flower pot traveling when it hits the street? (Give your answer in ft/sec and in mph, given that 1ft/sec = 15/22 mph)

The object in problem 3 falls off the same 400-foot tower. What would the acceleration due to gravity have to be to make it reach the ground in half the time?

An object is dropped from a 400-foot tower. When does it hit the ground and how fast is it going at the time of the impact?

A ball that is dropped from a window hits the ground in five seconds. How high is the window?

On the moon the acceleration due to gravity is 5 ft/sec^2. An astronaut jumps into the air with an initial upward velocity of 10 ft/sec. How high does he go? How long is the astronaut off the ground?

Galileo was the first person to show that the distance traveled by a body falling from rest is proportional to the square of the time it has traveled, and independent of the mass of the body. Derive this result from the fact that the acceleration due to gravity is a constant.

While attempting to understand the motion of bodies under gravity, Galileo stated that:
The time in which any space is traversed by a body starting at rest and uniformly accelerated is equal to the time in which that same space would be traversed by the same body moving at a uniform speed whose value is the mean of the highest velocity and the velocity just before acceleration began.
(a) Write Galileo's statement in symbols, defining all the symbols you use.
(b) Check Galileo's statement for a body dropped off a 100-foot building accelerating from rest under gravity until it hits the ground.
(c) Show why Galileo's statement is true in general.

An object thrown in the air on a planet in a distant galaxy is at height $s = -25t^2 + 72t + 40$ feet at time t seconds after it is thrown. What is the acceleration due to gravity on this planet? With what velocity was the object thrown? From what height?

An object is attached to a coiled spring which is suspended from the ceiling of a room. The function $h(t)$ gives the height of the object above the floor of the room at time, t. The graph of $h'(t)$ is given in Figure 6.33. Sketch a possible graph of $h(t)$.

The angular speed of a car engine increases from 1100 revs/min to 2500 revs/min in 6 sec.
(a) Assuming that it is constant, find the angular acceleration in revs/min^2.
(b) How many revolutions does the engine make in this time?

A helicopter rotor slows down at a constant rate from 350 revs/min to 260 revs/min in 1.5 minutes.
(a) Find the angular acceleration during this time interval. What are the units of this acceleration?
(b) Assuming the angular acceleration remains constant, how long does it take for the rotor to stop? (Measure time from the moment when speed was 350 revs/min.)
(c) How many revolutions does the rotor make between the time the angular speed was 350 revs/min and stopping?

An object is thrown vertically upward with a velocity of 80 ft/sec.
(a) Make a table showing its velocity every second.
(b) When does it reach its highest point? When does it hit the ground?
(c) Using your table, write left and right sums which under- and overestimate the height the object attains.
(d) Use antidifferentiation to find the greatest height it reaches.

A car, initially moving at 60 mph, has a constant deceleration and stops in a distance of 200 feet. What is its deceleration? (Give your answer in ft/sec^2. Note that 1 mph = 22/15 ft/sec.)

The birth rate, B, in births per hour, of a bacteria population is given in Figure 6.35. The curve marked D gives the death rate, in deaths per hour, of the same population.
(a) Explain what the shape of each of these graphs tells you about the population.
(b) Use the graphs to find the time at which the net rate of increase of the population is at a maximum.
(c) At time $t = 0$ the population has size N. Sketch the graph of the total number born by time t. Also sketch the graph of the number alive at time t. Estimate the time at which the population is a maximum.

Water flows at a constant rate into the left side of the W-shaped container in Figure 6.36. Sketch a graph of the height, H, of the water in the left side of the container as a function of time, t. The container starts empty.

A particle of mass, m, acted on by a force, F, moves in a straight line. Its acceleration, a, is given by Newton's Law: $F = ma$ The work, W, done by a constant force when the particle moves through a displacement, d, is $W = Fd$. The velocity, v, of the particle as a function of time, t, is given in Figure 6.37. What is the sign of the work done during each of the time intervals: $[0, t_1]$, $[t_1, t_2]$, $[t_2, t_3]$, $[t_3, t_4]$, $[t_2, t_4]$?

The concentration, C, in ng/ml, of a drug in the blood as a function of the time, t, in hours since the drug was administered is given by $C = 15te^{-0.2t}$. The area under the concentration curve is a measure of the overall effect of the drug on the body, called the bioavailability. Find the bioavailability of the drug between t = 0 and t = 3.

The voltage, V, in an electric circuit is given as a function of time, t, by $V = V_0 \cos(\omega t + \phi)$. Suppose each of the positive constants, V_0, ω, ϕ is increased (while the other two are held constant). What is the effect of each increase on the following quantities:
(a). The maximum value of V?
(b). The maximum value of dV/dt?
(c). The average value of V^2 over one period?

During a surge in the demand for electricity, the rate, r, at which energy is used can be approximated by $r = te^{-at}$, where t is the time in hours and a is a positive constant.
(a). Find the total energy, E, used in the first T hours. Give your answer as a function of a.
(b). What happens to E as $T \to \infty$?

The width, in feet, at various points along the fairway of a hole on a golf course is given in Figure 7.12. If one pound of fertilizer covers 200 square feet, estimate the amount of fertilizer needed to fertilize the fairway.

The curves y = sin x and y = cos x cross each other infinitely often. What is the area of the region bounded by these two curves between two consecutive crossings?

The solid obtained by rotating the region around the y-axis.

The solid obtained by rotating the region around the x-axis.

(a) A pie dish is 9 inches across the top, 7 inches across the bottom, and 3 inches deep. See Figure 8.26. Compute the volume of this dish.
(b) Make a rough estimate of the volume in cubic inches of a single cut-up apple, and estimate the number of apples that is needed to make an apple pie that fills this dish.

A 100 cm long gutter is made of three strips of metal, each 5 cm wide; Figure 8.27 shows a cross-section.
(a) Find the volume of water in the gutter when the depth is h cm.
(b) What is the maximum value of h?
(c) What is the maximum volume of water that the gutter can hold?
(d) If the gutter is filled with half the maximum volume of water, is the depth larger or smaller than half of the answer to part (b)? Explain how you can answer without any calculation.
(e) Find the depth of the water when the gutter contains half the maximum possible volume.

The design of boats is based on Archimedes' Principle, which states that the buoyant force on an object in water is equal to the weight of the water displaced. Suppose you want to build a sailboat whose hull is parabolic with cross section $y = ax^2$, where a is a constant. Your boat will have length L

and its maximum draft (the maximum vertical depth of any point of the boat beneath the water line) will be H. See Figure 8.28. Every cubic meter of water weighs 10,000 newtons. What is the maximum possible weight for your boat and cargo?

The circumference of a tree at different heights above the ground is given in the table below. Assume that all horizontal cross-sections of the tree are circles. Estimate the volume of the tree.

A bowl has the shape of the graph of $y = x^4$ between the points (1, 1) and (−1, 1) rotated about the y-axis. When the bowl contains water to a depth of h units, it flows out through a hole in the bottom at a rate (volume/time) proportional to $\sqrt{h}$, with constant of proportionality 6.
(a) Show that the water level falls at a constant rate.
(b) Find how long it takes to empty the bowl if it is originally full to the brim.

The hull of a boat has widths given by the following table. Reading across a row of the table gives widths at points 0, 10, …, 60 feet from the front to the back at a certain level below waterline. Reading down a column of the table gives widths at levels 0, 2, 4, 6, 8 feet below waterline at a certain distance from the front. Use the trapezoidal rule to estimate the volume of the hull below waterline.

The catenary $\cosh x = 1/2(e^x + e^{-x})$ represents the shape of a hanging cable. Find the exact length of this catenary between $x = -1$ and $x = 1$.

Set up an integral for the circumference of an ellipse with semi-major axis $a = 2$ and semi-minor axis $b = 1$. Evaluating this integral numerically (i.e., using a calculator or computer) poses a problem. Describe this problem. Formulate a way to solve it.

Find the area inside the spiral $r = \theta$ for $0 \le \theta \le 2\pi$.

Find the area between the two spirals $r = \theta$ and $r = 2\theta$ for $0 \le \theta \le 2\pi$.

Find the area inside the cardioid $r = 1 + \cos\theta$ for $0 \le \theta \le 2\pi$.

Find the area inside the circle $r = 1$ and outside the cardioid $r = 1 + \sin\theta$.

Find the total mass of the triangular region in figure 8.55, which has density $\delta(x) = 1 + x$ grams/cm^2.

The density of oil in a circular oil slick on the surface of the ocean at a distance r meters from the center of the slick is given by $\delta(r) = 50/(1 + r)$ kg/m^2.
(a) If the slick extends from $r = 0$ to $r = 10000$ m, find a Riemann sum approximating the total mass of oil in the slick.
(b) Find the exact value of the mass of oil in the slick by turning your sum into an integral and evaluating it.
(c) Within what distance r is half the oil of the slick contained?

The soot produced by a garbage incinerator spreads out in a circular pattern. The depth, H(r), in millimeters, of the soot deposited each month at a distance r kilometers from the incinerator is given by $H(r) = 0.115e^{-2r}$.
(a) Write a definite integral giving the total volume of soot deposited within 5 kilometers of the incinerator each month.
(b) Evaluate the integral you found in part (a), giving your answer in cubic meters.

A cardboard figure has the shape shown in figure 8.56. The region is bounded on the left by the line x=a, on the right by the line x=b, above by f(x), and below by g(x). If the density $\delta(x)$ gm/cm^2 varies only with x, find an expression for the total mass of the figure in terms of f(x), g(x), and $\delta(x)$.

The storage shed in Figure 8.57 is the shape of a half-cylinder of radius r and length l..
(a) What is the volume of the shed?
(b) The shed is filled with sawdust whose density (muss/unit volume) at any point is proportional to the distance of that point from the floor. Calculate the total mass of sawdust in the shed.

The following table gives the density *D* (in g/cm^3) of the Earth at a depth *x* km below the Earth's surface. The radius of the Earth is about 6370 km. Find an upper and lower bound for the Earth's mass such that the upper bound is less that twice the lower bound. Explain your reasoning, in particular, what assumptions have you made about the density?