Laurence D. Hoffmann, Gerald Bradley, Applied Calculus for Business, Economics, and the Social and Life Sciences, Tenth Edition

CONSUMER DEMAND In Exercises 57through 60, the demand function p =D(x) and the total cost function C(x) for a particular commodity are given in terms of the level of production x. In each case, find:
(a) The revenue R(x) and profit P(x).
(b) All values of x for which production of the commodity is profitable.

MANUFACTURING COST Suppose the total cost of manufacturing *q* units of a certain commodity is $C(q)$ thousand dollars, where
$C(q) = 0.01q^2 + 0.9q + 2$
a. Compute the cost of manufacturing 10 units.
b. Compute the cost of manufacturing the 10th unit.

DISTRIBUTION COST Suppose that the number of worker-hours required to distribute new telephone books to *x*% of the households in a certain rural community is given by the function

$$W(x) = \frac{600x}{300 - x}$$

a. What is the domain of the function *W*?
b. For what values of *x* does $W(x)$ have a practical interpretation in this context?
c. How many worker-hours were required to distribute new telephone books to the first 50% of the households?
d. How many worker-hours were required to distribute new telephone books to the entire community?
e. What percentage of the households in the community had received new telephone books by the time 150 worker-hours had been expended?

WORKER EFFICIENCY An efficiency study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8:00 A.M. will have assembled $f(x) = -x^3 + 6x^2 + 15x$ television sets *x* hours later.
a. How many sets will such a worker have assembled by 10:00 A.M.? [*Hint:* At 10:00 A.M., $x = 2$.]
b. How many sets will such a worker assemble between 9:00 and 10:00 A.M.?

IMMUNIZATION Suppose that during a nationwide program to immunize the population against a certain form of influenza, public health officials found that the cost of inoculating *x*% of the population was approximately $C(x) = \frac{150x}{200 - x}$ million dollars.
a. What is the domain of the function *C*?
b. For what values of *x* does $C(x)$ have a practical interpretation in this context?
c. What was the cost of inoculating the first 50% of the population?
d. What was the cost of inoculating the second 50% of the population?
e. What percentage of the population had been inoculated by the time 37.5 million dollars had been spent?

BLOOD FLOW Biologists have found that the speed of blood in an artery is a function of the distance of the blood from the artery's central axis. According to **Poiseuille's law,*** the speed (in centimeters per second) of blood that is *r* centimeters from the central axis of an artery is given by the function $S(r) = C(R^2 - r^2)$, where *C* is a constant and *R* is the radius of the artery. Suppose that for a certain artery, $C = 1.76 \times 10^5$ and $R = 1.2 \times 10^{-2}$ centimeters.
a. Compute the speed of the blood at the central axis of this artery.
b. Compute the speed of the blood midway between the artery's wall and central axis.

MANUFACTURING COST A manufacturer can produce digital recorders at a cost of $40 apiece. It is estimated that if the recorders are sold for *p* dollars apiece, consumers will buy 120 – *p* of them a month. Express the manufacturer's monthly profit as a function of price, graph this function, and use the graph to estimate the optimal selling price.

MANUFACTURING COST A manufacturer can produce tires at a cost of $20 apiece. It is estimated that if the tires are sold for *p* dollars apiece, consumers will buy 1,560 – 12*p* of them each month. Express the manufacturer's monthly profit as a function of price, graph this function, and use the graph to determine the optimal selling price. How many tires will be sold each month at the optimal price?

RENEWABLE RESOURCES The accompanying graph shows how the volume of lumber *V* in a tree varies with time *t* (the age of the tree). Use the graph to estimate the rate at which *V* is changing with respect to time when *t* = 30 years. What seems to be happening to the rate of change of *V* as *t* increases without bound (that is, in the "long run")?

POPULATION GROWTH The accompanying graph shows how a population *P* of fruit flies (*Drosophila*) changes with time *t* (days) during an experiment. Use the graph to estimate the rate at which the population is growing after 20 days and also after 36 days. At what time is the population growing at the greatest rate?

THERMAL INVERSION Air temperature usually decreases with increasing altitude. However, during the winter, thanks to a phenomenon called *thermal inversion,* the temperature of air warmed by the sun in mountains above a fog may rise above the freezing point, while the temperature at lower elevations remains near or below 0°C. Use the accompanying graph to estimate the rate at which temperature *T* is changing with respect to altitude *h* at an altitude of 1,000 meters and also at 2,000 meters.

PROFIT A manufacturer can produce digital recorders at a cost of $50 apiece. It is estimated that if the recorders are sold for *p* dollars apiece, consumers will buy *q* = 120 – *p* recorders each month.
a. Express the manufacturer's profit *P* as a function of *q*.
b. What is the average rate of profit obtained as the level of production increases from *q* = 0 to *q* = 20?
c. At what rate is profit changing when *q* = 20 recorders are produced? Is the profit increasing or decreasing at this level of production?

PROFIT A manufacturer determines that when *x* hundred units of a particular commodity are produced, the profit will be $P(x) = 4{,}000(15 - x)(x-2)$ dollars.
a. Find $P'(x)$.
b. Determine where $P'(x) = 0$. What is the significance of the level of production x_m where this occurs?

MANUFACTURING OUTPUT At a certain factory, it is determined that an output of *Q* units is to be expected when *L* worker-hours of labor are employed, where
$Q(L) = 3100\sqrt{L}$
a. Find the average rate of change of output as the labor employment changes from worker-hours to 3,100 worker-hours.
b. Use calculus to find the instantaneous rate of change of output with respect to labor level when L = 3025

COST OF PRODUCTION A business manager determines that the cost of producing *x* units of a particular commodity is *C* thousands of dollars, where $C(x) = 0.04x^2 + 5.1x + 40$
a. Find the average cost as the level of production changes from x = 10 to x = 11 units.

b. Use calculus to find the instantaneous rate of change of cost with respect to production level when x = 10 and compare with the average cost found in part (a). Is the cost increasing or decreasing when 10 units are being produced?

CONSUMER EXPENDITURE The demand for a particular commodity is given by $D(x) = -35x + 200$; that is, x units will be sold (demanded) at a price of $p = D(x)$ dollars per unit.
a. Consumer expenditure is the total amount of money consumers pay to buy x units. Express consumer expenditure E as a function of x.
b. Find the average change in consumer expenditure as x changes from x = 4 to x = 5.
c. Use calculus to find the instantaneous rate of change of expenditure with respect to x when x = 4. Is expenditure increasing or decreasing when x = 4?

UNEMPLOYMENT In economics, the graph in Figure 2.2 is called the **Phillips curve,** after A. W. Phillips, a New Zealander associated with the London School of Economics. Until Phillips published his ideas in the 1950s, many economists believed that unemployment and inflation were linearly related. Read an article on the Phillips curve (the source cited with Figure 2.2 would be a good place to start), and write a paragraph on the nature of unemployment in the U.S. economy.

BLOOD PRESSURE Refer to the graph of blood pressure as a function of time shown in Figure 2.9.
a. Estimate the average rate of change in blood pressure over the time periods [0.7, 0.75] and [0.75, 0.8]. Interpret your results.
b. Write a paragraph on the dynamics of the arterial pulse. Pages 131–136 in the reference given with Figure 2.9 is a good place to start, and there is an excellent list of annotated references to related topics on pp. 137–138.

ANIMAL BEHAVIOR Experiments indicate that when a flea jumps, its height (in meters) after t seconds is given by the function $H(t) = 4.4t - 4.9t^2$
a. Find $H'(t)$. At what rate is $H(t)$ changing after 1 second? Is it increasing or decreasing?
b. At what time is $H'(t) = 0$? What is the significance of this time?
c. When does the flea "land" (return to its initial height)? At what rate is $H(t)$ changing at this time? Is it increasing or decreasing?

VELOCITY A toy rocket rises vertically in such a way that t seconds after liftoff, it is $h(t) = -16t^2 + 200t$ feet above ground.
a. How high is the rocket after 6 seconds?
b. What is the average velocity of the rocket over the first 6 seconds of flight (between $t = 0$ and $t = 6$)?
c. What is the (instantaneous) velocity of the rocket at liftoff ($t = 0$)? What is its velocity after 40 seconds?

CARDIOLOGY A study conducted on a patient undergoing cardiac catheterization indicated that the diameter of the aorta was approximately Dmillimeters (mm) when the aortic pressure was p (mm of mercury), where
$D(p) = -0.0009p^2 + 0.13p + 17.81$
for $50 \le p \le 120$.
a. Find the average rate of change of the aortic diameter D as p changes from p = 60 to p = 61
b. Use calculus to find the instantaneous rate of change of diameter D with respect to aortic pressure p when p = 60. Is the pressure increasing or decreasing when p = 60?
c. For what value of p is the instantaneous rate of change of D with respect to p equal to 0? What is the significance of this pressure?

ANNUAL EARNINGS The gross annual earnings of a certain company were $A(t) = 0.1t^2 + 10t + 20$ thousand dollars t years after its formation in 2004.
a. At what rate were the gross annual earnings of the company growing with respect to time in 2008?
b. At what percentage rate were the gross annual earnings growing with respect to time in 2008?

WORKER EFFICIENCY An efficiency study of the morning shift at a certain factory indicates that an average worker who arrives on the job at 8:00 A.M. will have assembled
$f(x) = -x^3 + 6x^2 + 15x$ units x hours later.
a. Derive a formula for the rate at which the worker will be assembling units after x hours.
b. At what rate will the worker be assembling units at 9:00 A.M.?
c. How many units will the worker actually assemble between 9:00 A.M. and 10:00 A.M.?

EDUCATIONAL TESTING It is estimated that x years from now, the average SAT mathematics score of the incoming students at an eastern liberal arts college will be $f(x) = -6x + 582$.
a. Derive an expression for the rate at which the average SAT score will be changing with respect to time x years from now.
b. What is the significance of the fact that the expression in part (a) is a constant? What is the significance of the fact that the constant in part (a) is negative?

PUBLIC TRANSPORTATION After x weeks, the number of people using a new rapid transit system was approximately $N(x) = 6x^3 + 500x + 8{,}000$.
a. At what rate was the use of the system changing with respect to time after 8 weeks?
b. By how much did the use of the system change during the eighth week?

PROPERTY TAX Records indicate that x years after 2005, the average property tax on a threebedroom home in a certain community was
$T(x) = 20x^2 + 40x + 600$ dollars.
a. At what rate was the property tax increasing with respect to time in 2005?
b. By how much did the tax change between the years 2005 and 2009?

ADVERTISING A manufacturer of motorcycles estimates that if x thousand dollars is spent on advertising, then

$$M(x) = 2300 + \frac{125}{x} - \frac{517}{x^2} \qquad 3 \le x \le 18$$

cycles will be sold. At what rate will sales be changing when \$9,000 is spent on advertising? Are sales increasing or decreasing for this level of advertising expenditure?

POPULATION GROWTH It is projected that x months from now, the population of a certain town will be $P(x) = 2x + 4x^{3/2} + 5{,}000$.
a. At what rate will the population be changing with respect to time 9 months from now?
b. At what percentage rate will the population be changing with respect to time 9 months from now?

POPULATION GROWTH It is estimated that t years from now, the population of a certain town will be $P(t) = t^2 + 200t + 10{,}000$.
a. Express the percentage rate of change of the population as a function of t, simplify this function algebraically, and draw its graph.
b. What will happen to the percentage rate of change of the population in the long run (that is, as t grows very large)?

SPREAD OF AN EPIDEMIC A medical research team determines that t days after an epidemic begins $N(t) = 10t^3 + 5t + \sqrt{t}$, people will be infected, for $0 \le t \le 20$. At what rate is the infected population increasing on the ninth day?

SPREAD OF AN EPIDEMIC A disease is spreading in such a way that after t weeks, the number of people infected is

$N(t) = 5{,}175 - t^3(t - 8) \qquad 0 \le t \le 8$

a. At what rate is the epidemic spreading after 3 weeks?
b. Suppose health officials declare the disease to have reached epidemic proportions when the percentage rate of change of N is at least 25%.
Over what time period is this epidemic criterion satisfied?
c. Read an article on epidemiology and write a paragraph on how public health policy is related to the spread of an epidemic.

AIR POLLUTION An environmental study of a certain suburban community suggests that t years from now, the average level of carbon monoxide in the air will be $Q(t) = 0.05t^2 + 0.1t + 3.4$ parts per million.
a. At what rate will the carbon monoxide level be changing with respect to time 1 year from now?
b. By how much will the carbon monoxide level change this year?
c. By how much will the carbon monoxide level change over the next 2 years?

NEWSPAPER CIRCULATION It is estimated that t years from now, the circulation of a local newspaper will be $C(t) = 100t^2 + 400t + 5{,}000$.
a. Derive an expression for the rate at which the circulation will be changing with respect to time t years from now.
b. At what rate will the circulation be changing with respect to time 5 years from now? Will the circulation be increasing or decreasing at that time?
c. By how much will the circulation actually change during the sixth year?

SALARY INCREASES Your starting salary will be \$45,000, and you will get a raise of \$2,000 each year.
a. Express the percentage rate of change of your salary as a function of time and draw the graph.
b. At what percentage rate will your salary be increasing after 1 year?
c. What will happen to the percentage rate of change of your salary in the long run?

GROSS DOMESTIC PRODUCT The gross domestic product of a certain country is growing at a constant rate. In 1995 the GDP was 125 billion dollars, and in 2003 it was 155 billion dollars. If this trend continues, at what percentage rate will the GDP be growing in 2010?

ORNITHOLOGY An ornithologist determines that the body temperature of a certain species of bird fluctuates over roughly a 17-hour period according to the cubic formula
$T(t) = -68.07t^3 + 30.98t^2 + 12.52t + 37.1$ for $0 \le t \le 0.713$, where T is the temperature in degrees Celsius measured t days from the beginning of a period.
a. Compute and interpret the derivative $T'(t)$.
b. At what rate is the temperature changing at the beginning of the period ($t = 0$) and at the end of the period ($t = 0.713$)? Is the temperature increasing or decreasing at each of these times?
c. At what time is the temperature not changing (neither increasing nor decreasing)? What is the bird's temperature at this time? Interpret your result.

PHYSICAL CHEMISTRY According to Debye's formula in physical chemistry, the orientation polarization P of a gas satisfies

$$P = \frac{4}{3}\pi N\left(\frac{\mu^2}{3kT}\right)$$

where μ, k, and N are positive constants, and T is the temperature of the gas. Find the rate of change of P with respect to T.

COST MANAGEMENT A company uses a truck to deliver its products. To estimate costs, the manager models gas consumption by the function

$$G(x) = \frac{1}{250}\left(\frac{1200}{x} + x\right)$$

gal/mile, assuming that the truck is driven at a constant speed of x miles per hour, for $x \geq 5$. The driver is paid \$20 per hour to drive the truck 250 miles, and gasoline costs \$4 per gallon.
a. Find an expression for the total cost $C(x)$ of the trip.
b. At what rate is the cost $C(x)$ changing with respect to x when the truck is driven at 40 miles per hour? Is the cost increasing or decreasing at that speed?

MOTION OF A PROJECTILE A stone is dropped from a height of 144 feet.
a. When will the stone hit the ground?
b. With what velocity does it hit the ground?

MOTION OF A PROJECTILE You are standing on the top of a building and throw a ball vertically upward. After 2 seconds, the ball passes you on the way down, and 2 seconds after that, it hits the ground below.
a. What is the initial velocity of the ball?
b. How high is the building?
c. What is the velocity of the ball when it passes you on the way down?
d. What is the velocity of the ball as it hits the ground?

SPY STORY Our friend, the spy who escaped from the diamond smugglers in Chapter 1 (Problem 46 of Section 1.4), is on a secret mission in space. An encounter with an enemy agent leaves him with a mild concussion and temporary amnesia. Fortunately, he has a book that gives the formula for the motion of a projectile and the values of g for various heavenly bodies (32 ft/sec^2 on earth, 5.5 ft/sec^2 on the moon, 12 ft/sec^2 on Mars, and 28 ft/sec^2 on Venus). To deduce his whereabouts, he throws a rock vertically upward (from ground level) and notes that it reaches a maximum height of 37.5 ft and hits the ground 5 seconds after it leaves his hand. Where is he?

POLLUTION CONTROL It has been suggested that one way to reduce worldwide carbon dioxide (CO_2) emissions is to impose a single tax that would apply to all nations. The accompanying graph shows the relationship between different levels of the carbon tax and the percentage of reduction in CO_2 emissions.
a. What tax rate would have to be imposed to achieve a worldwide reduction of 50% in CO_2 emissions?
b. Use the graph to estimate the rate of change of the percentage reduction in CO_2 emissions when the tax rate is \$200 per ton.
c. Read an article on CO_2 emissions and write a paragraph on how public policy can be used to control air pollution.

DEMAND AND REVENUE The manager of a company that produces graphing calculators determines that when x thousand calculators are produced, they will all be sold when the price is

$$p(x) = \frac{1000}{0.3x^2 + 8}$$

dollars per calculator.

a. At what rate is demand $p(x)$ changing with respect to the level of production x when 3,000 ($x = 3$) calculators are produced?
b. The revenue derived from the sale of x thousand calculators is $R(x) = xp(x)$ thousand dollars. At what rate is revenue changing when 3,000 calculators are produced? Is revenue increasing or decreasing at this level of production?

SALES The manager of the Many Facets jewelry store models total sales by the function

$$S(t) = \frac{2000t}{4 + 0.3t}$$

where t is the time (years) since the year 2006 and S is measured in thousands of dollars.
a. At what rate were sales changing in the year 2008?
b. What happens to sales in the "long run" (that is, as $t \to \infty$)?

PROFIT Bea Johnson, the owner of the Bea Nice boutique, estimates that when a particular kind of perfume is priced at p dollars per bottle, she will sell

$$B(p) = \frac{500}{p+3} \qquad p \geq 5$$

bottles per month at a total cost of $C(p) = 0.2p^2 + 3p + 200$ dollars.
a. Express Bea's profit $P(p)$ as a function of the price p per bottle.
b. At what rate is the profit changing with respect to p when the price is \$12 per bottle? Is profit increasing or decreasing at that price?

ADVERTISING A company manufactures a "thin" DVD burner kit that can be plugged into personal computers. The marketing manager determines that t weeks after an advertising campaign begins, $P(t)$ percent of the potential market is aware of the burners, where

$$P(t) = 100\left[\frac{t^2 + 5t + 5}{t^2 + 10t + 30}\right]$$

a. At what rate is the market percentage $P(t)$ changing with respect to time after 5 weeks? Is the percentage increasing or decreasing at this time?
b. What happens to the percentage $P(t)$ in the "long run"; that is, as $t \to +\infty$? What happens to the rate of change of $P(t)$ as $t \to +\infty$?

BACTERIAL POPULATION A bacterial colony is estimated to have a population of

$$P(t) = \frac{24t + 10}{t^2 + 1}$$

million t hours after the introduction of a toxin.
a. At what rate is the population changing 1 hour after the toxin is introduced ($t = 1$)? Is the population increasing or decreasing at this time?
b. At what time does the population begin to decline?

POLLUTION CONTROL A study indicates that spending money on pollution control is effective up to a point but eventually becomes wasteful. Suppose it is known that when x million dollars is spent on controlling pollution, the percentage of pollution removed is given by

$$P(x) = \frac{100\sqrt{x}}{0.03x^2 + 9}$$

a. At what rate is the percentage of pollution removal $P(x)$ changing when 16 million dollars is spent? Is the percentage increasing or decreasing at this level of expenditure?
b. For what values of x is $P(x)$ increasing? For what values of x is $P(x)$ decreasing?

PHARMACOLOGY An oral painkiller is administered to a patient, and t hours later, the concentration of drug in the patient's bloodstream is given by

$$C(t) = \frac{2t}{3t^2 + 16}$$

a. At what rate $R(t)$ is the concentration of drug in the patient's bloodstream changing t hours after being administered? At what rate is $R(t)$ changing at time t?
b. At what rate is the concentration of drug changing after 1 hour? Is the concentration changing at an increasing or decreasing rate at this time?
c. When does the concentration of the drug begin to decline?
d. Over what time period is the concentration changing at a declining rate?

WORKER EFFICIENCY An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 A.M. will have produced $Q(t) = -t^3 + 8t^2 + 15t$ units t hours later.
a. Compute the worker's rate of production $R(t) = Q'(t)$.
b. At what rate is the worker's rate of production changing with respect to time at 9:00 A.M.?

POPULATION GROWTH It is estimated that t years from now, the population of a certain suburban community will be $P(t) = 20 - \frac{6}{t+1}$ thousand.
a. Derive a formula for the rate at which the population will be changing with respect to time t years from now.
b. At what rate will the population be growing 1 year from now?
c. By how much will the population actually increase during the second year?
d. At what rate will the population be growing 9 years from now?
e. What will happen to the rate of population growth in the long run?

VELOCITY An object moves along a straight line so that after t minutes, its distance from its starting point is $D(t) = 10t + \frac{5}{t+1} - 5$ meters.
a. At what velocity is the object moving at the end of 4 minutes?
b. How far does the object actually travel during the fifth minute?

ACCELERATION After t hours of an 8-hour trip, a car has gone $D(t) = 64t + \frac{10}{3}t^2 - \frac{2}{9}t^3$ kilometers.
a. Derive a formula expressing the acceleration of the car as a function of time.
b. At what rate is the velocity of the car changing with respect to time at the end of 6 hours? Is the velocity increasing or decreasing at this time?
c. By how much does the velocity of the car actually change during the seventh hour?

DRUG DOSAGE The human body's reaction to a dose of medicine can be modeled by a function of the form

$$F = \frac{1}{3}(KM^2 - M^3)$$

where K is a positive constant and M is the amount of medicine absorbed in the blood. The derivative $S = \frac{dF}{dM}$ can be thought of as a measure of the sensitivity of the body to the medicine.
a. Find the sensitivity S.
b. Find $\frac{dS}{dM} = \frac{d^2F}{dM^2}$ and give an interpretation of the second derivative.

BLOOD CELL PRODUCTION A biological model measures the production of a certain type of white blood cell *(granulocytes)* by the function

$$p(x) = \frac{Ax}{B + x^m}$$

where A and B are positive constants, the exponent m is positive, and x is the number of cells present.
a. Find the rate of production $p'(x)$.
b. Find $p''(x)$ and determine all values of x for which $p''(x) = 0$ (your answer will involve m).
c. Read an article on blood cell production and write a paragraph on how mathematical methods can be used to model such production. A good place to start is with the article, "Blood Cell Population Model, Dynamical Diseases, and Chaos" by W. B. Gearhart and M. Martelli, UMAP Module 1990, Arlington, MA: Consortium for Mathematics and Its Applications, Inc., 1991.

ACCELERATION If an object is dropped or thrown vertically, its height (in feet) after t seconds is $H(t) = -16t^2 + S_0 t + H_0$, where S_0 is the initial speed of the object and H_0 its initial height.
a. Derive an expression for the acceleration of the object.
b. How does the acceleration vary with time?
c. What is the significance of the fact that the answer to part (a) is negative?

ANNUAL EARNINGS The gross annual earnings of a certain company are $f(t) = \sqrt{10t^2 + t + 229}$ thousand dollars t years after its formation in January 2005.
a. At what rate will the gross annual earnings of the company be growing in January 2010?
b. At what percentage rate will the gross annual earnings be growing in January 2010?

MANUFACTURING COST At a certain factory, the total cost of manufacturing q units is $C(q) = 0.2q^2 + q + 900$ dollars. It has been determined that approximately $q(t) = t^2 + 100t$ units are manufactured during the first t hours of a production run. Compute the rate at which the total manufacturing cost is changing with respect to time 1 hour after production commences.

CONSUMER DEMAND An importer of Brazilian coffee estimates that local consumers will buy approximately $D(p) = \frac{4374}{p^2}$ pounds of the coffee per week when the price is p dollars per pound. It is also estimated that t weeks from now, the price of Brazilian coffee will be $p(t) = 0.02t^2 + 0.1t + 6$ dollars per pound.
a. At what rate will the demand for coffee be changing with respect to price when the price is \$9?
b. At what rate will the demand for coffee be changing with respect to time 10 weeks from now? Will the demand be increasing or decreasing at this time?

CONSUMER DEMAND When a certain commodity is sold for p dollars per unit, consumers will buy $D(p) = \frac{40000}{p}$ units per month. It is estimated that t months from now, the price of the commodity will be $p(t) = 0.4t^{3/2} + 6.8$ dollars per unit. At what percentage rate will the monthly demand for the commodity be changing with respect to time 4 months from now?

AIR POLLUTION It is estimated that t years from now, the population of a certain suburban community will be $p(t) = 20 - \frac{6}{t+1}$ thousand.
An environmental study indicates that the average daily level of carbon monoxide in the air will be $c(p) = 0.5\sqrt{p^2 + p + 58}$ parts per million when the population is p thousand.

a. At what rate will the level of carbon monoxide be changing with respect to population when the population is 18 thousand people?
b. At what rate will the carbon monoxide level be changing with respect to time 2 years from now? Will the level be increasing or decreasing at this time?

ANIMAL BEHAVIOR In a research paper, V. A. Tucker and K. Schmidt-Koenig demonstrated that a species of Australian parakeet (the Budgerigar) expends energy (calories per gram of mass per kilometer) according to the formula

$$E = \frac{1}{v}\left[0.074(v-35)^2 + 22\right]$$

where v is the bird's velocity (in km/hr). Find a formula for the rate of change of E with respect to velocity v.

MAMMALIAN GROWTH Observations show that the length L in millimeters (mm) from nose to tip of tail of a Siberian tiger can be estimated using the function $L = 0.25w^{2.6}$, where w is the weight of the tiger in kilograms (kg). Furthermore, when a tiger is less than 6 months old, its weight (kg) can be estimated in terms of its age A in days by the function $w = 3 + 0.21A$.
a. At what rate is the length of a Siberian tiger increasing with respect to its weight when it weighs 60 kg?
b. How long is a Siberian tiger when it is 100 days old? At what rate is its length increasing with respect to time at this age?

QUALITY OF LIFE A demographic study models the population p (in thousands) of a community by the function

$$p(Q) = 3Q^2 + 4Q + 200$$

where Q is a quality-of-life index that ranges from $Q = 0$ (extremely poor quality) to $Q = 10$ (excellent quality). Suppose the index varies with time in such a way that t years from now,

$$Q(t) = \frac{t^2 + 2t + 3}{2t + 1}$$

For $0 \le t \le 10$
a. What value of the quality-of-life index should be expected 4 years from now? What will be the corresponding population at this time?
b. At what rate is the population changing with respect to time 4 years from now? Is the population increasing or decreasing at this time?

WATER POLLUTION When organic matter is introduced into a body of water, the oxygen content of the water is temporarily reduced by oxidation. Suppose that t days after untreated sewage is dumped into a particular lake, the proportion of the usual oxygen content in the water of the lake that remains is given by the function

$$P(t) = 1 - \frac{12}{t+12} + \frac{144}{(t+12)^2}$$

a. At what rate is the oxygen proportion $P(t)$ changing after 10 days? Is the proportion increasing or decreasing at this time?
b. Is the oxygen proportion increasing or decreasing after 15 days?
c. If there is no new dumping, what would you expect to eventually happen to the proportion of oxygen? Use a limit to verify your conjecture.

PRODUCTION The number of units Q of a particular commodity that will be produced when L worker-hours of labor are employed is modeled by
$Q(L) = 300L^{1/3}$
Suppose that the labor level varies with time in such a way that t months from now, $L(t)$ workerhours

will be employed, where

$$L(t) = \sqrt{739 + 3t - t^2}$$

For $0 \le t \le 12$

a. How many worker-hours will be employed in producing the commodity 5 months from now? How many units will be produced at this time?

b. At what rate will production be changing with respect to time 5 months from now? Will production be increasing or decreasing at this time?

PRODUCTION The number of units Q of a particular commodity that will be produced with K thousand dollars of capital expenditure is modeled by

$$Q(K) = 500K^{2/3}$$

Suppose that capital expenditure varies with time in such a way that t months from now there will be $K(t)$ thousand dollars of capital expenditure, where

$$K(t) = \frac{2t^4 + 3t + 149}{t + 2}$$

a. What will be the capital expenditure 3 months from now? How many units will be produced at this time?

b. At what rate will production be changing with respect to time 5 months from now? Will production be increasing or decreasing at this time?

DEPRECIATION The value V (in thousands of dollars) of an industrial machine is modeled by

$$V(N) = \left(\frac{3N + 430}{N + 1}\right)^{2/3}$$

where N is the number of hours the machine is used each day. Suppose further that usage varies with time in such a way that

$$N(t) = \sqrt{t^2 - 10t + 45}$$

where t is the number of months the machine has been in operation.

a. How many hours per day will the machine be used 9 months from now? What will be the value of the machine at this time?

b. At what rate is the value of the machine changing with respect to time 9 months from now? Will the value be increasing or decreasing at this time?

INSECT GROWTH The growth of certain insects varies with temperature. Suppose a particular species of insect grows in such a way that the volume of an individual is

$$V(T) = 0.41(-0.01T^2 + 0.4T + 3.52) \ \text{cm}^3$$

when the temperature is T°C, and that its mass is m grams, where

$$m(V) = \frac{0.39V}{1 + 0.09V}$$

a. Find the rate of change of the insect's volume with respect to temperature.

b. Find the rate of change of the insect's mass with respect to volume.

c. When $T = 10$°C, what is the insect's volume? At what rate is the insect's mass changing with respect to temperature when $T = 10$°C?

COMPOUND INTEREST If \$10,000 is invested at an annual rate r (expressed as a decimal) compounded weekly, the total amount (principal P and interest) accumulated after 10 years is given by the formula

$$A = 10000\left(1 + \frac{r}{52}\right)^{520}$$

a. Find the rate of change of A with respect to r.
b. Find the percentage rate of change of A with respect to r when $r = 0.05$ (that is, 5%).

LEARNING When you first begin to study a topic or practice a skill, you may not be very good at it, but in time, you will approach the limits of your ability. One model for describing this behavior involves the function

$$T = aL\sqrt{L-b}$$

where T is the time required for a particular person to learn the items on a list of L items and a and b are positive constants.

a. Find the derivative $\frac{dT}{dL}$ and interpret it in terms of the learning model.
b. Read and discuss in one paragraph an article on how learning curves can be used to study worker productivity.

MARGINAL ANALYSIS In Exercises 1 through 6, C(x) is the total cost of producing x units of a particular commodity and p(x) is the price at which all x units will be sold. Assume p(x) and C(x) are in dollars.
(a) Find the marginal cost and the marginal revenue.
(b) Use marginal cost to estimate the cost of producing the fourth unit.
(c) Find the actual cost of producing the fourth unit.
(d) Use marginal revenue to estimate the revenue derived from the sale of the fourth unit.
(e) Find the actual revenue derived from the sale of the fourth unit.

MARGINAL ANALYSIS A manufacturer's total cost is $C(q) = 0.1q^3 - 0.5q^2 + 500q + 200$ dollars, where q is the number of units produced.
a. Use marginal analysis to estimate the cost of manufacturing the fourth unit.
b. Compute the actual cost of manufacturing the fourth unit.

MARGINAL ANALYSIS A manufacturer's total monthly revenue is $R(q) = 240q - 0.05q^2$ dollars when q units are produced and sold during the month. Currently, the manufacturer is producing 80 units a month and is planning to increase the monthly output by 1 unit.
a. Use marginal analysis to estimate the additional revenue that will be generated by the production and sale of the 81st unit.
b. Use the revenue function to compute the actual additional revenue that will be generated by the production and sale of the 81st unit.

MARGINAL ANALYSIS Suppose the total cost in dollars of manufacturing q units is $C(q) = 3q^2 + q + 500$.
a. Use marginal analysis to estimate the cost of manufacturing the 41st unit.
b. Compute the actual cost of manufacturing the 41st unit.

AIR POLLUTION An environmental study of a certain community suggests that t years from now, the average level of carbon monoxide in the air will be $Q(t) = 0.05t^2 + 0.1t + 3.4$ parts per million. By approximately how much will the carbon monoxide level change during the coming 6 months?

NEWSPAPER CIRCULATION It is projected that t years from now, the circulation of a local newspaper will be $C(t) = 100t^2 + 400t + 5{,}000$. Estimate the amount by which the circulation will increase during the next 6 months.

MANUFACTURING A manufacturer's total cost is $C(q) = 0.1q^3 - 0.5q^2 + 500q + 200$ dollars when the level of production is q units. The current level of production is 4 units, and the manufacturer is planning to increase this to 4.1 units. Estimate how the total cost will change as a result.

MANUFACTURING A manufacturer's total monthly revenue is $R(q) = 240q - 0.05q^2$ dollars when q units are produced during the month. Currently, the manufacturer is producing 80 units a month and is planning to decrease the monthly output by 0.65 unit. Estimate how the total monthly revenue will change as a result.

EFFICIENCY An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 A.M. will have assembled $f(x) = -x^3 + 6x^2 + 15x$ units x hours later. Approximately how many units will the worker assemble between 9:00 and 9:15 A.M.?

PRODUCTION At a certain factory, the daily output is $Q(K) = 600K^{1/2}$ units, where K denotes the capital investment measured in units of \$1,000. The current capital investment is \$900,000. Estimate the effect that an additional capital investment of \$800 will have on the daily output.

PRODUCTION At a certain factory, the daily output is $Q(L) = 60{,}000L^{1/3}$ units, where L denotes the size of the labor force measured in worker-hours. Currently 1,000 worker-hours of labor are used each day. Estimate the effect on output that will be produced if the labor force is cut to 940 worker-hours.

PROPERTY TAX A projection made in January of 2002 determined that x years later, the average property tax on a three-bedroom home in a certain community will be $T(x) = 60x^{3/2} + 40x + 1{,}200$ dollars. Estimate the percentage change by which the property tax will increase during the first half of the year 2010.

POPULATION GROWTH A 5-year projection of population trends suggests that t years from now, the population of a certain community will be $P(t) = -t^3 + 9t^2 + 48t + 200$ thousand.
a. Find the rate of change of population $R(t) = P'(t)$ with respect to time t.
b. At what rate does the population growth rate $R(t)$ change with respect to time?
c. Use increments to estimate how much $R(t)$ changes during the first month of the fourth year. What is the actual change in $R(t)$ during this time period?

PRODUCTION At a certain factory, the daily output is $Q = 3{,}000K^{1/2}L^{1/3}$ units, where K denotes the firm's capital investment measured in units of \$1,000 and L denotes the size of the labor force measured in worker-hours. Suppose that the current capital investment is \$400,000 and that 1,331 worker-hours of labor are used each day. Use marginal analysis to estimate the effect that an additional capital investment of \$1,000 will have on the daily output if the size of the labor force is not changed.

PRODUCTION The daily output at a certain factory is $Q(L) = 300L^{2/3}$ units, where L denotes the size of the labor force measured in worker-hours. Currently, 512 worker-hours of labor are used each day. Estimate the number of additional workerhours of labor that will be needed to increase daily output by 12.5 units.

MANUFACTURING A manufacturer's total cost is $C(q) = \frac{1}{6}q^3 + 642q + 400$ dollars when q units are produced. The current level of production is 4 units. Estimate the amount by which the manufacturer should decrease production to reduce the total cost by \$130.

GROWTH OF A CELL A certain cell has the shape of a sphere. The formulas $S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$ are used to compute the surface area and volume of the cell, respectively. Estimate the effect on S and V produced by a 1% increase in the radius r.

CARDIAC OUTPUT *Cardiac output* is the volume (cubic centimeters) of blood pumped by a person's heart each minute. One way of measuring cardiac output C is by Fick's formula

$$C = \frac{a}{x-b}$$

where x is the concentration of carbon dioxide in the blood entering the lungs from the right side of the heart and a and b are positive constants. If x is measured as $x = c$ with a maximum error of 3%, what is the maximum percentage error that can be incurred by measuring cardiac output with Fick's formula? (Your answer will be in terms of a, b, and c.)

MEDICINE A tiny spherical balloon is inserted into a clogged artery. If the balloon has an inner diameter of 0.01 millimeter (mm) and is made from material 0.0005 mm thick, approximately how much material is inserted into the artery? [*Hint:* Think of the amount of material as a change in volume ΔV, where $V = \frac{4}{3}\pi r^3$]

ARTERIOSCLEROSIS In *arteriosclerosis,* fatty material called plaque gradually builds up on the walls of arteries, impeding the flow of blood, which, in turn, can lead to stroke and heart attacks. Consider a model in which the carotid artery is represented as a circular cylinder with crosssectional radius $R = 0.3$ cm and length L. Suppose it is discovered that plaque 0.07 cm thick is distributed uniformly over the inner wall of the carotid artery of a particular patient. Use increments to estimate the percentage of the total volume of the artery that is blocked by plaque. [*Hint:* The volume of a cylinder of radius R and length L is $V = \pi R^2 L$. Does it matter that we have not specified the length L of the artery?]

BLOOD CIRCULATION In Exercise 57, Section 1.1, we introduced an important law attributed to the French physician, Jean Poiseuille. Another law discovered by Poiseuille says that the volume of the fluid flowing through a small tube in unit time under fixed pressure is given by the formula $V = kR^4$, where k is a positive constant and R is the radius of the tube. This formula is used in medicine to determine how wide a clogged artery must be opened to restore a healthy flow of blood.
a. Suppose the radius of a certain artery is increased by 5%. Approximately what effect does this have on the volume of blood flowing through the artery?
b. Read an article on the cardiovascular system and write a paragraph on the flow of blood.

EXPANSION OF MATERIAL The (linear) **thermal expansion coefficient** of an object is defined to be

$$\sigma = \frac{L'(T)}{L(T)}$$

where $L(T)$ is the length of the object when the temperature is T. Suppose a 50-meter span of a bridge is built of steel with $\sigma = 1.4 \times 10^{-5}$ per degree Celsius. Approximately how much will the length change during a year when the temperature varies from –20°C (winter) to 35°C (summer)?

RADIATION Stefan's law in physics states that a body emits radiant energy according to the formula $R(T) = kT^4$, where R is the amount of energy emitted from a surface whose temperature is T (in degrees kelvin) and k is a positive constant. Estimate the percentage change in R that results from a 2% increase in T.

MANUFACTURING The output at a certain plant is $Q = 0.08x^2 + 0.12xy + 0.03y^2$ units per day, where x is the number of hours of skilled labor used and y the number of hours of unskilled labor used. Currently, 80 hours of skilled labor and 200 hours of unskilled labor are used each day. Use calculus to estimate the change in unskilled labor that should be made to offset a 1-hour increase in skilled labor so that output will be maintained at its current level.

MANUFACTURING The output of a certain plant is $Q = 0.06x^2 + 0.14xy + 0.05y^2$ units per day, where x is the number of hours of skilled labor used and y is the number of hours of unskilled labor used. Currently, 60 hours of skilled labor and 300 hours of unskilled labor are used each day. Use calculus to estimate the change in unskilled labor that should be made to offset a 1-hour increase in skilled labor so that output will be maintained at its current level.

SUPPLY RATE When the price of a certain commodity is p dollars per unit, the manufacturer is willing to supply x hundred units, where $3p^2 - x^2 = 12$. How fast is the supply changing when the price is $4 per unit and is increasing at the rate of 87 cents per month?

DEMAND RATE When the price of a certain commodity is p dollars per unit, customers demand x hundred units of the commodity, where

$x^2 + 3px + p^2 = 79$

How fast is the demand x changing with respect to time when the price is $5 per unit and is decreasing at the rate of 30 cents per month?

DEMAND RATE When the price of a certain commodity is p dollars per unit, consumers demand x hundred units of the commodity, where

$75x^2 + 17p^2 = 5{,}300$

How fast is the demand x changing with respect to time when the price is $7 and is decreasing at the rate of 75 cents per month? (That is, $\frac{dp}{dt} = -0.75$.)

REFRIGERATION An ice block used for refrigeration is modeled as a cube of side s. The block currently has volume 125,000 cm3 and is melting at the rate of 1,000 cm3 per hour.

a. What is the current length s of each side of the cube? At what rate is s currently changing with respect to time t?

b. What is the current rate of change of the surface area S of the block with respect to time? [*Note:* A cube of side s has volume $V = s^3$ and surface area $S = 6s^2$.]

MEDICINE A tiny spherical balloon is inserted into a clogged artery and is inflated at the rate of 0.002π mm^3/min. How fast is the radius of the balloon growing when the radius is $R = 0.005$ mm? [*Note:* A sphere of radius R has volume $V = \frac{4}{3}\pi R^3$].

POLLUTION CONTROL An environmental study for a certain community indicates that there will be $Q(p) = p^2 + 4p + 900$ units of a harmful pollutant in the air when the population is p thousand people. If the population is currently 50,000 and is increasing at the rate of 1,500 per year, at what rate is the level of pollution increasing?

GROWTH OF A TUMOR A tumor is modeled as being roughly spherical, with radius R. If the radius of the tumor is currently $R = 0.54$ cm and is increasing at the rate of 0.13 cm per month, what is the corresponding rate of change of the volume $V = \frac{4}{3}\pi R^3$?

BOYLE'S LAW Boyle's law states that when gas is compressed at constant temperature, the pressure P and volume V of a given sample satisfy the equation $PV = C$, where C is constant. Suppose that at a certain time the volume is 40 in.^{3}, the pressure is 70 lb/in.^{2}, and the volume is increasing at the rate of 12 in.^{3}/sec. How fast is the pressure changing at this instant? Is it increasing or decreasing?

METABOLIC RATE The *basal metabolic rate* is the rate of heat produced by an animal per unit time. Observations indicate that the basal metabolic rate of a warm-blooded animal of mass w kilograms (kg) is given by
$M = 70w^{3/4}$ kilocalories per day
a. Find the rate of change of the metabolic rate of an 80-kg cougar that is gaining mass at the rate of 0.8 kg per day.
b. Find the rate of change of the metabolic rate of a 50-kg ostrich that is losing mass at the rate of 0.5 kg per day.

SPEED OF A LIZARD Herpetologists have proposed using the formula $s = 1.1w^{0.2}$ to estimate the maximum sprinting speed s (meters per second) of a lizard of mass w (grams). At what rate is the maximum sprinting speed of an 11-gram lizard increasing if the lizard is growing at the rate of 0.02 grams per day?

PRODUCTION At a certain factory, output is given by $Q = 60K^{1/3}L^{2/3}$ units, where K is the capital investment (in thousands of dollars) and L is the size of the labor force, measured in workerhours. If output is kept constant, at what rate is capital investment changing at a time when $K = 8$, $L = 1{,}000$, and L is increasing at the rate of 25 worker-hours per week?
[*Note:* Output functions of the general form $Q = AK^{\alpha}L^{1-\alpha}$, where A and α are constants with $0 \le \alpha \le 1$, are called **Cobb-Douglas production functions.** Such functions appear in examples and exercises throughout this text, especially in Chapter 7.]

WATER POLLUTION A circular oil slick spreads in such a way that its radius is increasing at the rate of 20 ft/hr. How fast is the area of the slick changing when the radius is 200 feet?

A 6-foot-tall man walks at the rate of 4 ft/sec away from the base of a street light 12 feet above the ground. At what rate is the length of his shadow changing when he is 20 feet away from the base of the light?

CHEMISTRY In an *adiabatic* chemical process, there is no net change (gain or loss) of heat. Suppose a container of oxygen is subjected to such a process. Then if the pressure on the oxygen is P and its volume is V, it can be shown that $PV^{1.4} = C$, where C is a constant. At a certain time, $V = 5$ m^3, $P = 0.6$ kg/m^2, and P is increasing at 0.23 kg/m^2 per sec. What is the rate of change of V? Is V increasing or decreasing?

MANUFACTURING At a certain factory, output Q is related to inputs x and y by the equation
$Q = 2x^3 + 3x^2y^2 + (1 + y)^3$
If the current levels of input are $x = 30$ and $y = 20$, use calculus to estimate the change in input y that should be made to offset a decrease of 0.8 unit in input x so that output will be maintained at its current level.

LUMBER PRODUCTION To estimate the amount of wood in the trunk of a tree, it is reasonable to assume that the trunk is a cutoff cone (see the figure). If the upper radius of the trunk is r, the lower radius is R, and the height is H, the volume of the wood is given by

$$V = \frac{\pi}{3}H(R^2 + rR + r^2)$$

Suppose r, R, and H are increasing at the respective rates of 4 in/yr, 5 in/yr, and 9 in/yr. At what rate is V increasing at the time when $r = 2$ feet, $R = 3$ feet, and $H = 15$ feet?

BLOOD FLOW One of Poiseuille's laws (see Exercise 57, Section 1.1) says that the speed of blood flowing under constant pressure in a blood vessel at a distance r from the center of the vessel is given by

$$v = \frac{K}{L}(R^2 - r^2)$$

where K is a positive constant, R is the radius of the vessel, and L is the length of the vessel. Suppose the radius R and length L of the vessel change with time in such a way that the speed of blood flowing at the center is unaffected; that is, v does not change with time. Show that in this case, the relative rate of change of L with respect to time must be twice the relative rate of change of R.

MANUFACTURING At a certain factory, output Q is related to inputs u and v by the equation

$$Q = 3u^2 + \frac{2u + 3v}{(u + v)^2}$$

If the current levels of input are $u = 10$ and $v = 25$, use calculus to estimate the change in input v that should be made to offset a decrease of 0.7 unit in input u so that output will be maintained at its current level.

POPULATION GROWTH Suppose that a 5-year projection of population trends suggests that t years from now, the population of a certain community will be P thousand, where
$P(t) = -t^3 + 9t^2 + 48t + 200$.
a. At what rate will the population be growing 3 years from now?
b. At what rate will the rate of population growth be changing with respect to time 3 years from now?

RAPID TRANSIT After x weeks, the number of people using a new rapid transit system was approximately $N(x) = 6x^3 + 500x + 8{,}000$.
a. At what rate was the use of the system changing with respect to time after 8 weeks?
b. By how much did the use of the system change during the eighth week?

39. PRODUCTION It is estimated that the weekly output at a certain plant is $Q(x) = 50x^2 + 9{,}000x$ units, where x is the number of workers employed at the plant. Currently there are 30 workers employed at the plant.
a. Use calculus to estimate the change in the weekly output that will result from the addition of 1 worker to the force.
b. Compute the actual change in output that will result from the addition of 1 worker.

POPULATION It is projected that t months from now, the population of a certain town will be $P(t) = 3t + 5t^{3/2} + 6{,}000$. At what percentage rate will the population be changing with respect to time 4 months from now?

PRODUCTION At a certain factory, the daily output is $Q(L) = 20{,}000L^{1/2}$ units, where L denotes the size of the labor force measured in worker-hours. Currently 900 worker-hours of labor are used each day. Use calculus to estimate the effect on output that will be produced if the labor force is cut to 885 worker-hours.

GROSS DOMESTIC PRODUCT The gross domestic product of a certain country was $N(t) = t^2 + 6t + 300$ billion dollars t years after 2000. Use calculus to predict the percentage change in the GDP during the second quarter of 2008.

POLLUTION The level of air pollution in a certain city is proportional to the square of the population. Use calculus to estimate the percentage by which the air pollution level will increase if the population increases by 5%.

AIDS EPIDEMIC In its early phase, specifically the period 1984–1990, the AIDS epidemic could be modeled* by the cubic function
$C(t) = -170.36t^3 + 1{,}707.5t^2 + 1{,}998.4t + 4{,}404.8$

for $0 \le t \le 6$, where C is the number of reported cases t years after the base year 1984.
a. Compute and interpret the derivative $C'(t)$.
b. At what rate was the epidemic spreading in the year 1984?
c. At what percentage rate was the epidemic spreading in 1984? In 1990?

POPULATION DENSITY The formula $D = 36\ m\ +1.14$ is sometimes used to determine the ideal population density D (individuals per square kilometer) for a large animal of mass m kilograms (kg).
a. What is the ideal population density for humans, assuming that a typical human weighs about 70 kg?
b. The area of the United States is about 9.2 million square kilometers. What would the population of the United States have to be for the population density to be ideal?
c. Consider an island of area 3,000 km^2. Two hundred animals of mass m =30 kg are brought to the island, and t years later, the population is given by
$P(t) = 0.43t^2 + 13.37t + 200$
How long does it take for the ideal population density to be reached? At what rate is the population changing when the ideal density is attained?

BACTERIAL GROWTH The population P of a bacterial colony t days after observation begins is modeled by the cubic function
$P(t) = 1.035t^3 + 103.5t^2 + 6{,}900t + 230{,}000$
a. Compute and interpret the derivative $P'(t)$.
b. At what rate is the population changing after 1 day? After 10 days?
c. What is the initial population of the colony? How long does it take for the population to double? At what rate is the population growing at the time it doubles?

PRODUCTION The output at a certain factory is $Q(L) = 600L^{2/3}$ units, where L is the size of the labor force. The manufacturer wishes to increase output by 1%. Use calculus to estimate the percentage increase in labor that will be required.

PRODUCTION The output Q at a certain factory is related to inputs x and y by the equation
$Q = x^3 + 2xy^2 + 2y^3$
If the current levels of input are $x = 10$ and $y = 20$, use calculus to estimate the change in input y that should be made to offset an increase of 0.5 in input x so that output will be maintained at its current level.

BLOOD FLOW Physiologists have observed that the flow of blood from an artery into a small capillary is given by the formula
$F = kD^2\sqrt{A - C}$
where D is the diameter of the capillary, A is the pressure in the artery, C is the pressure in the capillary, and k is a positive constant.
a. By how much is the flow of blood F changing with respect to pressure C in the capillary if A and D are kept constant? Does the flow increase or decrease with increasing C?
b. What is the percentage rate of change of flow F with respect to A if C and D are kept constant?

POLLUTION CONTROL It is estimated that t years from now, the population of a certain suburban community will be $p(t) = 10 - \dfrac{20}{(t+1)^2}$ thousand. An environmental study indicates that the average daily level of carbon monoxide in the air will be $c(p) = 0.8\sqrt{p^2 + p + 139}$ units when the population is p thousand. At what percentage rate will the level of carbon monoxide be changing with respect to time 1 year from now?

You measure the radius of a circle to be 12 cm and use the formula $A = \pi r^2$ to calculate the area. If your measurement of the radius is accurate to within 3%, how accurate is your calculation of the area?

Estimate what will happen to the volume of a cube if the length of each side is decreased by 2%. Express your answer as a percentage.

PRODUCTION The output at a certain factory is $Q = 600K^{1/2}L^{1/3}$ units, where K denotes the capital investment and L is the size of the labor force. Estimate the percentage increase in output that will result from a 2% increase in the size of the labor force if capital investment is not changed.

BLOOD FLOW The speed of blood flowing along the central axis of a certain artery is $S(R) = 1.8 \times 10^5 R^2$ centimeters per second, where R is the radius of the artery. A medical researcher measures the radius of the artery to be 1.2×10^{-2} centimeter and makes an error of 5×10^{-4} centimeter. Estimate the amount by which the calculated value of the speed of the blood will differ from the true speed if the incorrect value of the radius is used in the formula.

AREA OF A TUMOR You measure the radius of a spherical tumor to be 1.2 cm and use the formula $S = 4\pi r^2$ to calculate the surface area. If your measurement of the radius r is accurate to within 3%, how accurate is your calculation of the area?

CARDIOVASCULAR SYSTEM One model of the cardiovascular system relates $V(t)$, the stroke volume of blood in the aorta at a time t during systole (the contraction phase), to the pressure $P(t)$ in the aorta during systole by the equation

$$V(t) = [C_1 + C_2 P(t)]\left(\frac{3t^2}{T^2} - \frac{2t^3}{T^3}\right)$$

where C_1 and C_2 are positive constants and T is the (fixed) time length of the systole phase.* Find a relationship between the rates $\frac{dV}{dt}$ and $\frac{dP}{dt}$.

CONSUMER DEMAND When electric toasters are sold for p dollars apiece, local consumers will buy $D(p) = \frac{32670}{2p+1}$ toasters. It is estimated that t months from now, the unit price of the toasters will be $p(t) = 0.04t^{3/2} + 44$ dollars. Compute the rate of change of the monthly demand for toasters with respect to time 25 months from now. Will the demand be increasing or decreasing?

At noon, a truck is at the intersection of two roads and is moving north at 70 km/hr. An hour later, a car passes through the same intersection, traveling east at 105 km/hr. How fast is the distance between the car and truck changing at 2 P.M.?

POPULATION GROWTH It is projected that t years from now, the population of a certain suburban community will be thousand. By approximately what percentage will the population grow during the next quarter year?

WORKER EFFICIENCY An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 A.M. will have produced $Q(t) = -t^3 + 9t^2 + 12t$ units t hours later.

a. Compute the worker's rate of production $R(t) = Q'(t)$.
b. At what rate is the worker's rate of production changing with respect to time at 9:00 A.M.?
c. Use calculus to estimate the change in the worker's rate of production between 9:00 and 9:06 A.M.

d. Compute the actual change in the worker's rate of production between 9:00 and 9:06 A.M.

TRAFFIC SAFETY A car is traveling at 88 ft/sec when the driver applies the brakes to avoid hitting a child. After t seconds, the car is $s = 88t - 8t^2$ feet from the point where the brakes were applied. How long does it take for the car to come to a stop, and how far does it travel before stopping?

CONSTRUCTION MATERIAL Sand is leaking from a bag in such a way that after t seconds, there are

$$S(t) = 50\left(1 - \frac{t^2}{15}\right)^3$$

pounds of sand left in the bag.

a. How much sand was originally in the bag?
b. At what rate is sand leaking from the bag after 1 second?
c. How long does it take for all of the sand to leak from the bag? At what rate is the sand leaking from the bag at the time it empties?

INFLATION It is projected that t months from now, the average price per unit for goods in a certain sector of the economy will be P dollars, where $P(t) = 1 - t^3 + 7t^2 + 200t + 300$.

a. At what rate will the price per unit be increasing with respect to time 5 months from now?
b. At what rate will the rate of price increase be changing with respect to time 5 months from now?
c. Use calculus to estimate the change in the rate of price increase during the first half of the sixth month.
d. Compute the actual change in the rate of price increase during the first half of the sixth month.

PRODUCTION COST At a certain factory, approximately $q(t) = t^2 + 50t$ units are manufactured during the first t hours of a production run, and the total cost of manufacturing q units is $C(q) = 0.1q^2 + 10q + 400$ dollars. Find the rate at which the manufacturing cost is changing with respect to time 2 hours after production commences.

PRODUCTION COST It is estimated that the monthly cost of producing x units of a particular commodity is $C(x) = 0.06x + 3x^{1/2} + 20$ hundred dollars. Suppose production is decreasing at the rate of 11 units per month when the monthly production is 2,500 units. At what rate is the cost changing at this level of production?

Estimate the largest percentage error you can allow in the measurement of the radius of a sphere if you want the error in the calculation of its surface area using the formula $S = 4\pi r^2$ to be no greater than 8%.

A soccer ball made of leather 1/8 inch thick has an inner diameter of 8.5 inches. Estimate the volume of its leather shell. [*Hint:* Think of the volume of the shell as a certain change ΔV in volume.]

A car traveling north at 60 mph and a truck traveling east at 45 mph leave an intersection at the same time. At what rate is the distance between them changing 2 hours later?

A child is flying a kite at a height of 80 feet above her hand. If the kite moves horizontally at a constant speed of 5 ft/sec, at what rate is string being paid out when the kite is 100 feet away from the child?

A person stands at the end of a pier 8 feet above the water and pulls in a rope attached to a buoy. If the rope is hauled in at the rate of 2 ft/min, how fast is the buoy moving in the water when it is 6 feet from the pier?

A 10-foot-long ladder leans against the side of a wall. The top of the ladder is sliding down the wall at the rate of 3 ft/sec. How fast is the foot of the ladder moving away from the building when the top is 6 feet above the ground?

A lantern falls from the top of a building in such a way that after t seconds, it is $h(t) = 150 - 16t^2$ feet above ground. A woman 5 feet tall originally standing directly under the lantern sees it start to fall and walks away at the constant rate of 5 ft/sec. How fast is the length of the woman's shadow changing when the lantern is 10 feet above the ground?

A baseball diamond is a square, 90 feet on a side. A runner runs from second base to third at 20 ft/sec. How fast is the distance s between the runner and home base changing when he is 15 feet from third base?

MANUFACTURING COST Suppose the total manufacturing cost C at a certain factory is a function of the number q of units produced, which in turn is a function of the number t of hours during which the factory has been operating.

a. What quantity is represented by the derivative $\frac{dC}{dq}$? In what units is this quantity measured?

b. What quantity is represented by the derivative $\frac{dq}{dt}$?In what units is this quantity measured?

c. What quantity is represented by the product $\frac{dC}{dq}\frac{dq}{dt}$? In what units is this quantity measured?

An object projected from a point P moves along a straight line. It is known that the velocity of the object is directly proportional to the product of the time the object has been moving and the distance it has moved from P. It is also known that at the end of 5 seconds, the object is 20 feet from P and is moving at the rate of 4 ft/sec. Find the acceleration of the object at this time (when $t = 5$).

Find all the points (x, y) on the graph of the function $y = 4x^2$ with the property that the tangent to the graph at (x, y) passes through the point (2, 0).

Suppose y is a linear function of x; that is, $y = mx + b$. What will happen to the percentage rate of change of y with respect to x as x increases without bound? Explain.

Find an equation for the tangent line to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0).

Chapter 3

In Exercises 9 through 22 find the intervals of increase and decrease for the given function.
$f(x) = x^2 - 4x + 5$

In Exercises 23 through 34 determine the critical numbers of the given function and classify each critical point as a relative maximum, a relative minimum, or neither.
$f(x) = 3x^4 - 8x^3 + 6x^2 + 2$

In Exercises 35 through 44, use calculus to sketch the graph of the given function.
$f(x) = x^3 - 3x^2$

AVERAGE COST The total cost of producing x units of a certain commodity is $C(x)$ thousand dollars, where

$C(x) = x^3 - 20x^2 + 179x + 242$

a. Find $A'(x)$, where A(x) = C(x)/x is the average cost function.

b. For what values of x is $A(x)$ increasing? For what values is it decreasing?

c. For what level of production x is average cost minimized? What is the minimum average cost?

MARGINAL ANALYSIS The total cost of producing x units of a certain commodity is given by $C(x) = \sqrt{5x+2} + 3$ Sketch the cost curve and find the marginal cost. Does marginal cost increase or decrease with increasing production?

MARGINAL ANALYSIS Let $p = (10 - 3x)^2$ for $0 \le x \le 3$ be the price at which x hundred units of a certain commodity will be sold, and let $R(x) = xp(x)$ be the revenue obtained from the sale of the x units. Find the marginal revenue $R'(x)$ and sketch the revenue and marginal revenue curves on the same graph. For what level of production is revenue maximized?

PROFIT UNDER A MONOPOLY To produce x units of a particular commodity, a monopolist has a total cost of $C(x) = 2x^2 + 3x + 5$ and total revenue of $R(x) = xp(x)$, where $p(x) = 5 - 2x$ is the price at which the x units will be sold. Find the profit function $P(x) = R(x) - C(x)$ and sketch its graph. For what level of production is profit maximized?

MEDICINE The concentration of a drug t hours after being injected into the arm of a patient is given by $C(t) = \dfrac{0.15t}{t^2 + 0.81}$

Sketch the graph of the concentration function. When does the maximum concentration occur?

POLLUTION CONTROL Commissioners of a certain city determine that when x million dollars are spent on controlling pollution, the percentage of pollution removed is given by

$$P(x) = \frac{100\sqrt{x}}{0.04x^2 + 12}$$

a. Sketch the graph of $P(x)$.

b. What expenditure results in the largest percentage of pollution removal?

ADVERTISING A company determines that if x thousand dollars are spent on advertising a certain product, then $S(x)$ units of the product will be sold, where

$S(x) = -2x^3 + 27x^2 + 132x + 207 \qquad 0 \le x \le 17$

a. Sketch the graph of $S(x)$.

b. How many units will be sold if nothing is spent on advertising?

c. How much should be spent on advertising to maximize sales? What is the maximum sales level?

MORTGAGE REFINANCING When interest rates are low, many homeowners take the opportunity to refinance their mortgages. As rates start to rise, there is often a flurry of activity as latecomers rush in to refinance while they still can do so profitably. Eventually, however, rates reach a level where refinancing begins to wane. Suppose in a certain community, there will be $M(r)$ thousand refinanced mortgages when the 30-year fixed mortgage rate is r %, where

$$M(r) = \frac{1 + 0.05r}{1 + 0.004r^2} \qquad 1 \le r \le 8$$

a. For what values of r is $M(r)$ increasing?

b. For what interest rate r is the number of refinanced mortgages maximized? What is this maximum number?

POPULATION DISTRIBUTION A demographic study of a certain city indicates that $P(r)$ hundred people live r miles from the civic center, where

$$P(r) = \frac{5(3r+1)}{r^2+r+2}$$

a. What is the population at the city center?
b. For what values of *r* is *P*(*r*) increasing? For what values is it decreasing?
c. At what distance from the civic center is the population largest? What is this largest population?

GROSS DOMESTIC PRODUCT The graph shows the consumption of the baby boom generation, measured as a percentage of total GDP (gross domestic product) during the time period 1970–1997.
a. At what years do relative maxima occur?
b. At what years do relative minima occur?
c. At roughly what rate was consumption increasing in 1987?
d. At roughly what rate was consumption decreasing in 1972?

DEPRECIATION The value *V* (in thousands of dollars) of an industrial machine is modeled by

$$V(N) = \left(\frac{3N+430}{N+1}\right)^{2/3}$$

where *N* is the number of hours the machine is used each day. Suppose further that usage varies with time in such a way that

$$N(t) = \sqrt{t^2 - 10t + 61}$$

where *t* is the number of months the machine has been in operation.
a. Over what time interval is the value of the machine increasing? When is it decreasing?
b. At what time *t* is the value of the machine the largest? What is this maximum value?

FISHERY MANAGEMENT The manager of a fishery determines that *t* weeks after 300 fish of a particular species are released into a pond, the average weight of an individual fish (in pounds) for the first 10 weeks will be $w(t) = 3 + t - 0.05t^2$. He further determines that the proportion of the fish that are still alive after *t* weeks is given by

$$p(t) = \frac{31}{31+t}$$

a. The expected yield *Y*(*t*) of the fish after *t* weeks is the total weight of the fish that are still alive. Express *Y*(*t*) in terms of *w*(*t*) and *p*(*t*) and sketch the graph of *Y*(*t*) for $0 \le t \le 10$.
b. When is the expected yield *Y*(*t*) the largest? What is the maximum yield?

FISHERY MANAGEMENT Suppose for the situation described in Exercise 65, it costs the fishery $C(t) = 50 + 1.2t$ hundred dollars to maintain and monitor the pond for *t* weeks after the fish are released, and that each fish harvested after *t* weeks can be sold for \$2.75 per pound.
a. If all fish that remain alive in the pond after *t* weeks are harvested, express the profit obtained by the fishery as a function of *t*.
b. When should the fish be harvested in order to maximize profit? What is the maximum profit?

Sketch a graph of a function that has all of the following properties:
$f'(0) = f'(1) = f'(2) = 0$

In Exercises 5 through 12, determine where the graph of the given function is concave upward and concave downward. Find the coordinates of all inflection points.
$f(x) = x^3 + 3x^2 + x + 1$

In Exercises 13 through 26, determine where the given function is increasing and decreasing, and where its graph is concave up and concave down. Find the relative extrema and inflection points and sketch the graph of the function.
$f(x) = x^3 + 3x^2 + 1$

In Exercises 27 through 38 use the second derivative test to find the relative maxima and minima of the given function.
$f(x) = x^4 - 2x^2 + 3$

In Exercises 39 through 42, the second derivative $f''(x)$ of a function is given. In each case, use this information to determine where the graph of $f(x)$ is concave upward and concave downward and find all values of x for which an inflection point occurs.
$f''(x) = x^2(x - 3)(x - 1)$

In Exercises 43 through 46, the first derivative $f'(x)$ of a certain function $f(x)$ is given. In each case,
(a) Find intervals on which f is increasing and decreasing.
(b) Find intervals on which the graph of f is concave up and concave down.
(c) Find the x coordinates of the relative extrema and inflection points of f .
(d) Sketch a possible graph for $f(x)$.
$f'(x) = x^2 - 4x$

MARGINAL ANALYSIS The cost of producing *x* units of a commodity per week is
$C(x) = 0.3x^3 - 5x^2 + 28x + 200$
a. Find the marginal cost $C'(x)$ and sketch its graph along with the graph of $C(x)$ on the same coordinate plane.
b. Find all values of *x* where $C''(x) = 0$. How are these levels of production related to the graph of the marginal cost?

MARGINAL ANALYSIS The profit obtained from producing *x* thousand units of a particular commodity each year is $P(x)$ dollars, where
$P(x) = -x^{9/2} + 90x^{7/2} - 5000$
a. Find the marginal profit $P'(x)$, and determine all values of *x* such that $P'(x) = 0$.
b. Sketch the graph of marginal profit along with the graph of $P(x)$ on the same coordinate plane.
c. Find $P''(x)$, and determine all values of *x* such that $P''(x)$. How are these levels of production related to the graph of marginal profit?

SALES A company estimates that if *x* thousand dollars are spent on marketing a certain product, then $S(x)$ units of the product will be sold each month, where $S(x) = -x^3 + 33x^2 + 60x + 1{,}000$
a. How many units will be sold if no money is spent on marketing?
b. Sketch the graph of $S(x)$. For what value of *x* does the graph have an inflection point? What is the significance of this marketing expenditure?

WORKER EFFICIENCY An efficiency study of the morning shift (from 8:00 A.M. to 12:00 noon) at a factory indicates that an average worker who arrives on the job at 8:00 A.M. will have produced *Q* units *t* hours later, where
$$Q(t) = -t^3 + \frac{9}{2}t^2 + 15t$$
a. At what time during the morning is the worker performing most efficiently?
b. At what time during the morning is the worker performing least efficiently?

POPULATION GROWTH A 5-year projection of population trends suggests that *t* years from now, the population of a certain community will be $P(t) = -t^3 + 9t^2 + 48t + 50$ thousand.
a. At what time during the 5-year period will the population be growing most rapidly?
b. At what time during the 5-year period will the population be growing least rapidly?
c. At what time is the rate of population growth changing most rapidly?

ADVERTISING The manager of the Footloose sandal company determines that t months after initiating an advertising campaign, $S(t)$ hundred pairs of sandals will be sold, where

$$S(t)=\frac{3}{t+2}-\frac{12}{(t+2)^2}+5$$

a. Find S ′(t) and S ″(t).
b. At what time will sales be maximized? What is the maximum level of sales?
c. The manager plans to terminate the advertising campaign when the sales rate is minimized. When does this occur? What are the sales level and sales rate at this time?

HOUSING STARTS Suppose that in a certain community, there will be $M(r)$ thousand new houses built when the 30-year fixed mortgage rate is r%, where

$$M(r)=\frac{1+0.02r}{1+0.009r^2}$$

a. Find M ′(r) and M ″(r).
b. Sketch the graph of the construction function $M(r)$.
c. At what rate of interest r is the rate of construction of new houses minimized?

GOVERNMENT SPENDING During a recession, Congress decides to stimulate the economy by providing funds to hire unemployed workers for government projects. Suppose that t months after the stimulus program begins, there are $N(t)$ thousand people unemployed, where
$N(t) = -t^3 + 45t^2 + 408t + 3{,}078$
a. What is the maximum number of unemployed workers? When does the maximum level of unemployment occur?
b. In order to avoid overstimulating the economy (and inducing inflation), a decision is made to terminate the stimulus program as soon as the rate of unemployment begins to decline. When does this occur? At this time, how many people are unemployed?

SPREAD OF A DISEASE An epidemiologist determines that a particular epidemic spreads in such a way that t weeks after the outbreak, N hundred new cases will be reported, where

$$N(t)=\frac{5t}{12+t^2}$$

a. Find N ′(r) and N ″(r).
b. At what time is the epidemic at its worst? What is the maximum number of reported new cases?
c. Health officials declare the epidemic to be under control when the rate of reported new cases is minimized. When does this occur? What number of new cases will be reported at that time?

THE SPREAD OF AN EPIDEMIC Let $Q(t)$ denote the number of people in a city of population N_0 who have been infected with a certain disease t days after the beginning of an epidemic. Studies indicate that the rate $R(Q)$ at which an epidemic spreads is jointly proportional to the number of people who have contracted the disease and the number who have not, so $R(Q) = kQ(N_0 - Q)$. Sketch the graph of the rate function, and interpret your graph. In particular, what is the significance of the highest point on the graph of $R(Q)$?

SPREAD OF A RUMOR The rate at which a rumor spreads through a community of P people is jointly proportional to the number of people N who have heard the rumor and the number who have not. Show that the rumor is spreading most rapidly when half the people have heard it.

POPULATION GROWTH Studies show that when environmental factors impose an upper bound on the possible size of a population $P(t)$, the population often tends to grow in such a way that the percentage rate of change of $P(t)$ satisfies

$$\frac{100P'(t)}{P(t)} = A - BP(t)$$

where A and B are positive constants. Where does the graph of $P(t)$ have an inflection point? What is the significance of this point? (Your answer will be in terms of A and B.)

TISSUE GROWTH Suppose a particular tissue culture has area $A(t)$ at time t and a potential maximum area M. Based on properties of cell division, it is reasonable to assume that the area A grows at a rate jointly proportional to $\sqrt{A(t)}$ and $M - A(t)$; that is,

$$\frac{dA}{dt} = k\sqrt{A(t)}[M - A(t)]$$

where k is a positive constant.

a. Let $R(t) = A'(t)$ be the rate of tissue growth. Show that $R'(t) = 0$ when $A(t) = M/3$.

b. Is the rate of tissue growth greatest or least when $A(t) = M/3$? [*Hint:* Use the first derivative test or the second derivative test.]

c. Based on the given information and what you discovered in part (a), what can you say about the graph of $A(t)$?

Water is poured at a constant rate into the vase shown in the accompanying figure. Let $h(t)$ be the height of the water in the vase at time t (assume the vase is empty when $t = 0$). Sketch a rough graph of the function $h(t)$. In particular, what happens when the water level reaches the neck of the vase?

Let $f(x) = x^4 + x$. Show that even though $f''(0) = 0$ the graph of f has neither a relative extremum nor an inflection point where $x = 0$. Sketch the graph of $f(x)$.

Use calculus to show that the graph of the quadratic function $y = ax^2 + bx + c$ is concave upward if a is positive and concave downward if a is negative.

AVERAGE COST The total cost of producing x units of a particular commodity is C thousand dollars, where $C(x) = 3x^2 + x + 48$, and the average cost is

$$A(x) = \frac{C(x)}{x} = 3x + 1 + \frac{48}{x}$$

a. Find all vertical and horizontal asymptotes of the graph of $A(x)$.

b. Note that as x gets larger and larger, the term $\frac{48}{x}$ in $A(x)$ gets smaller and smaller. What does this say about the relationship between the average cost curve $y = A(x)$ and the line $y = 3x + 1$?

c. Sketch the graph of $A(x)$, incorporating the result of part (b) in your sketch. [*Note:* The line $y = 3x + 1$ is called an *oblique* (or *slant*) *asymptote* of the graph.]

INVENTORY COST A manufacturer estimates that if each shipment of raw materials contains x units, the total cost in dollars of obtaining and storing the year's supply of raw materials will be

$$C(x) = 2x + \frac{80000}{x}$$

a. Find all vertical and horizontal asymptotes of the graph of $C(x)$.

b. Note that as x gets larger and larger, the term in $C(x)$ gets smaller and smaller. What does this say about the relationship between the cost curve $y = C(x)$ and the line $y = 2x$?

c. Sketch the graph of $C(x)$, incorporating the result of part (b) in your sketch. [*Note:* The line $y = 2x$ is called an *oblique* (or *slant*) *asymptote* of the graph.]

DISTRIBUTION COST The number of workerhours W required to distribute new telephone books to x% of the households in a certain community is modeled by the function

$$W(x) = \frac{200x}{100 - x}$$

a. Sketch the graph of $W(x)$.
b. Suppose only 1,500 worker-hours are available for distributing telephone books. What percentage of households do not receive new books?

PRODUCTION A business manager determines that t months after production begins on a new product, the number of units produced will be P million per month, where

$$P(t) = \frac{t}{(t+1)^2}$$

a. Find P ′(t) and P ″(t).
b. Sketch the graph of $P(t)$.
c. What happens to production in the long run.

SALES A company estimates that if x thousand dollars are spent on the marketing of a certain product, then $Q(x)$ thousand units of the product will be sold, where

a. Sketch the graph of the sales function $Q(x)$.
b. For what marketing expenditure x are sales maximized? What is the maximum sales level?
c. For what value of x is the sales rate minimized?

CONCENTRATION OF DRUG A patient is given an injection of a particular drug at noon, and samples of blood are taken at regular intervals to determine the concentration of drug in the patient's system. It is found that the concentration increases at an increasing rate with respect to time until 1 P.M., and for the next 3 hours, continues to increase but at a decreasing rate until the peak concentration is reached at 4 P.M. The concentration then decreases at a decreasing rate until 5 P.M., after which it decreases at an increasing rate toward zero. Sketch a possible graph for the concentration of drug $C(t)$ as a function of time.

BACTERIAL POPULATION The population of a bacterial colony increases at an increasing rate for 1 hour, after which it continues to increase but at a rate that gradually decreases toward zero. Sketch a possible graph for the population $P(t)$ as a function of time t.

EPIDEMIOLOGY Epidemiologists studying a contagious disease observe that the number of newly infected people increases at an increasing rate during the first 3 years of the epidemic. At that time, a new drug is introduced, and the number of infected people declines at a decreasing rate. Two years after its introduction, the drug begins to lose effectiveness. The number of new cases continues to decline for 1 more year but at an increasing rate before rising again at an increasing rate. Draw a possible graph for the number of new cases $N(t)$ as a function of time.

ADOPTION OF TECHNOLOGY Draw a possible graph for the percentage of households adopting a new type of consumer electronic technology if the percentage grows at an increasing rate for the first 2 years, after which the rate of increase declines, with the market penetration of the technology eventually approaching 90%.

EXPERIMENTAL PSYCHOLOGY To study the rate at which animals learn, a psychology student performed an experiment in which a rat was sent repeatedly through a laboratory maze. Suppose the time required for the rat to traverse the maze on the nth trial was approximately

$$f(n) = 3 + \frac{12}{n}$$

a. Graph the function $f(n)$.
b. What portion of the graph is relevant to the practical situation under consideration?
c. What happens to the graph as n increases without bound? Interpret your answer in practical terms.

AVERAGE TEMPERATURE A researcher models the temperature T (in degrees Celsius) during the time period from 6 A.M. to 6 P.M. in a certain city by the function

$$T(t) = -\frac{1}{36}t^3 + \frac{1}{8}t^2 + \frac{7}{3}t - 2 \qquad \text{for } 0 \le t \le 12$$

where t is the number of hours after 6 A.M.
a. Sketch the graph of $T(t)$.
b. At what time is the temperature the greatest? What is the highest temperature of the day?

IMMUNIZATION During a nationwide program to immunize the population against a new strain of influenza, public health officials determined that the cost of inoculating x% of the susceptible population would be approximately

$$C(x) = \frac{1.7x}{100 - x}$$

million dollars.
a. Sketch the graph of the cost function $C(x)$.
b. Suppose 40 million dollars are available for providing immunization. What percentage of the susceptible population will not be inoculated?

POLITICAL POLLING A poll commissioned by a politician estimates that t days after she comes out in favor of a controversial bill, the percentage of her constituency (those who support her at the time she declares her position on the bill) that still supports her is given by

$$S(t) = \frac{100(t^2 - 3t + 25)}{t^2 + 7t + 25}$$

The vote is to be taken 10 days after she announces her position.
a. Sketch the graph of $S(t)$ for $0 \le t \le 10$.
b. When is her support at its lowest level? What is her minimum support level?
c. The derivative $S'(t)$ may be thought of as an approval rate. Is her approval rate positive or negative when the vote is taken? Is the approval rate increasing or decreasing at this time? Interpret your results.

ADVERTISING A manufacturer of motorcycles estimates that if x thousand dollars are spent on advertising, then for x _ 0, cycles will be sold.

$$M(x) = 2300 + \frac{125}{x} - \frac{500}{x^2}$$

a. Sketch the graph of the sales function $M(x)$.
b. What level of advertising expenditure results in maximum sales? What is the maximum sales level?

COST MANAGEMENT A company uses a truck to deliver its products. To estimate costs, the manager models gas consumption by the function

$$G(x) = \frac{1}{2000}\left(\frac{800}{x} + 5x\right)$$

gal/mile, assuming that the truck is driven at a constant speed of x miles per hour for $x \ge 5$. The driver is paid \$18 per hour to drive the truck 400 miles, and gasoline costs \$4.25 per gallon. Highway regulations require $30 \le x \le 65$.

a. Find an expression for the total cost $C(x)$ of the trip. Sketch the graph of $C(x)$ for the legal speed interval $30 \le x \le 65$.
b. What legal speed will minimize the total cost of the trip? What is the minimal total cost?

Find constants A, B, and C so that the function $f(x) = Ax^3 + Bx^2 + C$ will have a relative extremum at (2, 11) and an inflection point at (1, 5). Sketch the graph of f.

MAXIMUM PROFIT AND MINIMUM AVERAGE COST In Exercises 17 through 22, you are given the price p(q) at which q units of a particular commoditycan be sold and the total cost C(q) of producing the q units. In each case:
(a) Find the revenue function R(q), the profit function P(q), the marginal revenue R ′(q), and marginal cost C ′(q). Sketch the graphs of P(q), R ′(q), and C ′(q) on the same coordinate axes and determine the level of production q where P(q) is maximized.
(b) Find the average cost A(q) = C(q)/q and sketch the graphs of A(q), and the marginal cost C ′(q) on the same axes. Determine the level of production q at which A(q) is minimized.

ELASTICITY OF DEMAND In Exercises 23 through 28, compute the elasticity of demand for the given demand function D(p) and determine whether the demand is elastic, inelastic, or of unit elasticity at the indicated price p.
$D(p) = -1.3p + 10;\ p = 4$

At what point does the tangent to the curve $y = 2x^3 - 3x^2 + 6x$ have the smallest slope? What is the slope of the tangent at this point?

AVERAGE PROFIT A manufacturer estimates that when q units of a certain commodity are produced, the profit obtained is $P(q)$ thousand dollars, where
$P(q) = -2q^2 + 68q - 128$
a. Find the average profit and the marginal profit functions.
b. At what level of production $\bar{q}$ is average profit equal to marginal profit?
c. Show that average profit is maximized at the level of production found in part (b).
d. On the same set of axes, graph the relevant portions of the average and marginal profit functions.

MARGINAL ANALYSIS A manufacturer estimates that if x units of a particular commodity are produced, the total cost will be $C(x)$ dollars, where
$C(x) = x^3 - 24x^2 + 350x + 338$
a. At what level of production will the marginal cost $C'(x)$ be minimized?
b. At what level of production will the average cost $A(x) = \dfrac{C(x)}{x}$ be minimized?

GROUP MEMBERSHIP A national consumers' association determines that x years after its founding in 1993, it will have $P(x)$ members, where
$P(x) = 100(2x^3 - 45x^2 + 264x)$
a. At what time between 1995 and 2008 was the membership largest? Smallest?
b. What were the largest and smallest membership levels between 1995 and 2008?

BROADCASTING An all-news radio station has made a survey of the listening habits of local residents between the hours of 5:00 P.M. and midnight. The survey indicates that the percentage of the local adult population that is tuned in to the station x hours after 5:00 P.M. is
$$f(x) = \frac{1}{8}(-2x^3 + 27x^2 - 108x + 240)$$
a. At what time between 5:00 P.M. and midnight are the most people listening to the station?

What percentage of the population is listening at this time?
b. At what time between 5:00 P.M. and midnight are the fewest people listening? What percentage of the population is listening at this time?

LEARNING In a learning model, two responses (A and B) are possible for each of the series of observations. If there is a probability p of getting response A in any individual observation, the probability of getting response A exactly n times in a series of m observations is $F(p) = p^n(1-p)^{m-n}$. The **maximum likelihood estimate** is the value of p that maximizes $F(p)$ for $0 \le p \le 1$. For what value of p does this occur?

GROWTH OF A SPECIES More on codling moths. The percentage of codling moths that survive the pupa stage at a given temperature T (degrees Celsius) is modeled by the formula $P(T) = -1.42T^2 + 68T - 746$ for $20 \le T \le 30$ Find the temperatures at which the greatest and smallest percentage of moths survive.

BLOOD CIRCULATION Poiseuille's law asserts that the speed of blood that is r centimeters from the central axis of an artery of radius R is $S(r) = c(R^2 - r^2)$, where c is a positive constant. Where is the speed of the blood greatest?

POLITICS A poll indicates that x months after a particular candidate for public office declares her candidacy, she will have the support of $S(x)$ percent of the voters, where

$$S(x) = \frac{1}{29}(-x^3 + 6x^2 + 63x + 1080) \quad \text{for } 0 \le x \le 12$$

If the election is held in November, when should the politician announce her candidacy? Should she expect to win if she needs at least 50% of the vote?

ELASTICITY OF DEMAND When a particular commodity is priced at p dollars per unit, consumers demand q units, where p and q are related by the equation $q^2 + 3pq = 22$.
a. Find the elasticity of demand for this commodity.
b. For a unit price of \$3, is the demand elastic, inelastic, or of unit elasticity?

ELASTICITY OF DEMAND When an electronics store prices a certain brand of stereo at p hundred dollars per set, it is found that q sets will be sold each month, where $q^2 + 2p^2 = 41$.
a. Find the elasticity of demand for the stereos.
b. For a unit price of $p = 4$ (\$400), is the demand elastic, inelastic, or of unit elasticity?

DEMAND FOR ART An art gallery offers 50 prints by a famous artist. If each print in the limited edition is priced at p dollars, it is expected that $q = 500 - 2p$ prints will be sold.
a. What limitations are there on the possible range of the price p?
b. Find the elasticity of demand. Determine the values of p for which the demand is elastic, inelastic, and of unit elasticity.
c. Interpret the results of part (b) in terms of the behavior of the total revenue as a function of unit price p.
d. If you were the owner of the gallery, what price would you charge for each print? Explain the reasoning behind your decision.

DEMAND FOR AIRLINE TICKETS An airline determines that when a round-trip ticket between Los Angeles and San Francisco costs p dollars ($0 \le p \le 160$), the daily demand for tickets is $q = 256 - 0.01p^2$.
a. Find the elasticity of demand. Determine the values of p for which the demand is elastic, inelastic, and of unit elasticity.
b. Interpret the results of part (a) in terms of the behavior of the total revenue as a function of

unit price p.
c. What price would you advise the airline to charge for each ticket? Explain your reasoning.

ORNITHOLOGY According to the results of Tucker and Schmidt-Koenig, the energy expended by a certain species of parakeet is given by

$$E(v) = \frac{1}{v}[0.074(v-35)^2 + 22]$$

where v is the bird's velocity (in km/hr).
a. What velocity minimizes energy expenditure?
b. Read an article on how mathematical methods can be used to study animal behavior, and write a paragraph on whether you think such methods are valid. You may wish to begin with the reference cited in this problem.

SPEED OF FLIGHT In a model† developed by C. J. Pennycuick, the power P required by a bird to maintain flight is given by the formula

$$P = \frac{w^2}{2\rho S v} + \frac{1}{2}\rho A v^3$$

where v is the relative speed of the bird, w is the weight, ρ is the density of air, and S and A are positive constants associated with the bird's size and shape. What relative speed v will minimize the power required by the bird?

PRODUCTION CONTROL A toy manufacturer produces an inexpensive doll (Floppsy) and an expensive doll (Moppsy) in units of x hundreds and y hundreds, respectively. Suppose that it is possible to produce the dolls in such a way that

$$y = \frac{82-10x}{10-x}$$

for $0 \le x \le 8$ and that the company receives *twice* as much for selling a Moppsy doll as for selling a Floppsy doll. Find the level of production (both x and y) for which the total revenue derived from selling these dolls is maximized. You can assume that the company sells every doll it produces.

VOTING PATTERN After a presidential election, the proportion $h(p)$ of seats in the House of Representatives won by the party of the winning presidential candidate may be modeled by the "cube rule"

$$h(p) = \frac{p^3}{p^3 + (1-p)^3}$$

where p is the proportion of the popular vote received by the winning presidential candidate.
a. Find $h'(p)$ and $h''(p)$.
b. Sketch the graph of $h(p)$.
c. In 1964, the Democrat Lyndon Johnson received 61% of the popular vote. What percentage of seats in the House does the cube rule predict should have gone to Democrats? (They actually won 72%.)
d. For the most part, the cube rule has been extremely accurate for presidential elections since 1900. Use the Internet to research the actual proportions that occurred during these elections and write a paragraph on your findings.

WORKER EFFICIENCY An efficiency study of the morning shift at a certain factory indicates that an average worker who is on the job at 8.00 A.M. will have assembled $f(x) = -x^3 + 6x^2 + 15x$ units x hours later. The study indicates further that after a 15-minute coffee break the worker can assemble

$g(x) = -\frac{1}{3}x^3 + x^2 + 23x$ units in x hours.

Determine the time between 8.00 A.M. and noon at which a 15-minute coffee break should be scheduled so that the worker will assemble the maximum number of units by lunchtime at 12:15 P.M.

NATIONAL CONSUMPTION Assume that total national consumption is given by a function $C(x)$, where x is the total national income. The derivative $C'(x)$ is called the **marginal propensity to consume.** Then $S = x - C$ represents total national savings, and $S'(x)$ is called **marginal propensity to save.** Suppose the consumption function is $C(x) = 8 - 0.8x - 0.8\sqrt{x}$. Find the marginal propensity to consume, and determine the value of x that results in the smallest total savings.

SENSITIVITY TO DRUGS Body reaction to drugs is often modeled* by an equation of the form

$$R(D) = D^2\left(\frac{C}{2} - \frac{D}{3}\right)$$

where D is the dosage and C (a constant) is the maximum dosage that can be given. The rate of change of $R(D)$ with respect to D is called the **sensitivity.**

a. Find the value of D for which the sensitivity is the greatest. What is the greatest sensitivity? (Express your answer in terms of C.)

b. What is the reaction (in terms of C) when the dosage resulting in greatest sensitivity is used?

AERODYNAMICS In designing airplanes, an important feature is the so-called drag factor; that is, the retarding force exerted on the plane by the air. One model measures drag by a function of the form

$$F(v) = Av^2 + \frac{B}{v^2}$$

where v is the velocity of the plane and A and B are constants. Find the velocity (in terms of A and B) that minimizes $F(v)$. Show that you have found the minimum rather than a maximum.

ELECTRICITY When a resistor of R ohms is connected across a battery with electromotive force E volts and internal resistance r ohms, a current of I amperes will flow, generating P watts of power, where

$$I = \frac{E}{r+R} \text{ and } P = I^2R$$

Assuming r is constant, what choice of R results in maximum power?

SURVIVAL OF AQUATIC LIFE It is known that a quantity of water that occupies 1 liter at 0°C will occupy

$$V(t) = \left(\frac{-6.8}{10^8}\right)T^3 + \left(\frac{8.5}{10^6}\right)T^2 - \left(\frac{6.4}{10^5}\right)T + 1$$

liters when the temperature is T°C, for $0 \le T \le 30$.

a. Use a graphing utility to graph $V(T)$ for $0 \le T \le 10$. The density of water is maximized when $V(T)$ is minimized. At what temperature does this occur?

b. Does the answer to part (a) surprise you? It should. Water is the only common liquid whose maximum density occurs *above* its freezing point (0°C for water). Read an article on the survival of aquatic life during the winter and then write a paragraph on how the property of water examined in this problem is related to such survival.

BLOOD PRODUCTION A useful model for the production $p(x)$ of blood cells involves a function of the form

$$p(x) = \frac{Ax}{B + x^m}$$

where x is the number of cells present, and A, B, and m are positive constants.

a. Find the rate of blood production $R(x) = p'(x)$ and determine where $R(x) = 0$.
b. Find the rate at which $R(x)$ is changing with respect to x and determine where $R'(x) = 0$.
c. If $m > 1$, does the nonzero critical number you found in part (b) correspond to a relative maximum or a relative minimum? Explain.

AMPLITUDE OF OSCILLATION In physics, it can be shown that a particle forced to oscillate in a resisting medium has amplitude $A(r)$ given by

$$A(r) = \frac{1}{(1-r^2)^2 + kr^2}$$

where r is the ratio of the forcing frequency to the natural frequency of oscillation and k is a positive constant that measures the damping effect of the resisting medium. Show that $A(r)$ has exactly one positive critical number. Does it correspond to a relative maximum or a relative minimum? Can anything be said about the *absolute* extrema of $A(r)$?

RESPIRATION Physiologists define the flow F of air in the trachea by the formula $F = SA$, where S is the speed of the air and A is the area of a cross section of the trachea.
a. Assume the trachea has a circular cross section with radius r. Use the formula for the speed of air in the trachea during a cough given in Example 3.4.3 to express air flow F in terms of r.
b. Find the radius r for which the flow is greatest.

MARGINAL ANALYSIS Suppose $q > 0$ units of a commodity are produced at a total cost of $C(q)$ dollars and an average cost of $A(q) = \frac{C(q)}{q}$. In this section, we showed that $q = q_c$ satisfies $A'(qc) = 0$ if and only if $C'(q_c) = A(q_c)$; that is, when marginal cost equals average cost. The purpose of this problem is to show that $A(q)$ is *minimized* when $q = q_c$.
a. Generally speaking, the cost of producing a commodity increases at an increasing rate as more and more goods are produced. Using this economic principle, what can be said about the sign of $C''(q)$ as q increases?
b. Show that $A''(q_c) > 0$ if and only if $C''(qc) > 0$. Then use part (a) to argue that average cost $A(q)$ is minimized when $q = q_c$.

ELASTICITY AND REVENUE Suppose the demand for a certain commodity is given by $q = b - ap$, where a and b are positive constants, and $0 \le p \le \frac{b}{a}$.
a. Express elasticity of demand as a function of p.
b. Show that the demand is of unit elasticity at the midpoint $p = \frac{b}{2a}$ of the interval $0 \le p \le \frac{b}{a}$
c. For what values of p is the demand elastic? Inelastic?

ELASTICITY Suppose that the demand equation for a certain commodity is $q = \frac{a}{p^m}$, where a and m are positive constants. Show that the elasticity of demand is equal to $-m$ for all values of p. Interpret this result.

MARGINAL ANALYSIS Let $R(x)$ be the revenue obtained from the production and sale of x units of a commodity, and let $C(x)$ be the total cost of producing the x units. Show that the ratio $Q(x) = \frac{R(x)}{C(x)}$ is optimized when the relative rate of change of revenue equals the relative rate of change of cost. Would you expect this optimum to be a maximum or a minimum?

What number exceeds its square by the largest amount? [*Hint:* Find the number x that maximizes $f(x) = x - x^2$.]

What number is exceeded by its square root by the largest amount?

Find two positive numbers whose sum is 50 and whose product is as large as possible.

Find two positive numbers x and y whose sum is 30 and are such that xy^2 is as large as possible.

RETAIL SALES A store has been selling a popular computer game at the price of $40 per unit, and at this price, players have been buying 50 units per month. The owner of the store wishes to raise the price of the game and estimates that for each $1 increase in price, three fewer units will be sold each month. If each unit costs the store $25, at what price should the game be sold to maximize profit?

RETAIL SALES A bookstore can obtain a certain gift book from the publisher at a cost of $3 per book. The bookstore has been offering the book at a price of $15 per copy and, at this price, has been selling 200 copies a month. The bookstore is planning to lower its price to stimulate sales and estimates that for each $1 reduction in the price, 20 more books will be sold each month. At what price should the bookstore sell the book to generate the greatest possible profit?

AGRICULTURAL YIELD A Florida citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted on the same acreage. How many trees should the grower plant to maximize the total yield?

HARVESTING Farmers can get $8 per bushel for their potatoes on July 1, and after that, the price drops by 8 cents per bushel per day. On July 1, a farmer has 80 bushels of potatoes in the field and estimates that the crop is increasing at the rate of 1 bushel per day. When should the farmer harvest the potatoes to maximize revenue?

PROFIT A baseball card store can obtain Mel Schlabotnic rookie cards at a cost of $5 per card. The store has been offering the cards at $10 apiece and, at this price, has been selling 25 cards per month. The store is planning to lower the price to stimulate sales and estimates that for each 25-cent reduction in the price, 5 more cards will be sold each month. At what price should the cards be sold in order to maximize total monthly profit?

PROFIT A manufacturer has been selling flashlights at $6 apiece, and at this price, consumers have been buying 3,000 flashlights per month. The manufacturer wishes to raise the price and estimates that for each $1 increase in the price, 1,000 fewer flashlights will be sold each month. The manufacturer can produce the flashlights at a cost of $4 per flashlight. At what price should the manufacturer sell the flashlights to generate the greatest possible profit?

FENCING A city recreation department plans to build a rectangular playground having an area of 3,600 square meters and surround it by a fence. How can this be done using the least amount of fencing?

FENCING There are 320 yards of fencing available to enclose a rectangular field. How should this fencing be used so that the enclosed area is as large as possible?

Prove that of all rectangles with a given perimeter, the square has the largest area.

Prove that of all rectangles with a given area, the square has the smallest perimeter.

A rectangle is inscribed in a right triangle, as shown in the accompanying figure. If the triangle has sides of length 5, 12, and 13, what are the dimensions of the inscribed rectangle of greatest area?

A triangle is positioned with its hypotenuse on a diameter of a circle, as shown in the accompanying figure. If the circle has radius 4, what are the dimensions of the triangle of greatest area?

CONSTRUCTION COST A carpenter has been asked to build an open box with a square base. The sides of the box will cost \$3 per square meter, and the base will cost \$4 per square meter. What are the dimensions of the box of greatest volume that can be constructed for \$48?

CONSTRUCTION COST A closed box with a square base is to have a volume of 250 cubic meters. The material for the top and bottom of the box costs \$2 per square meter, and the material for the sides costs \$1 per square meter. Can the box be constructed for less than \$300?

SPY STORY It is noon, and the spy is back from space (see Exercise 74 in Section 2.2) and driving a jeep through the sandy desert in the tiny principality of Alta Loma. He is 32 kilometers from the nearest point on a straight paved road. Down the road 16 kilometers is an abandoned power plant where a group of rival spies are holding captive his superior, code name "N." If the spy doesn't arrive with a ransom by 12:50 P.M., the bad guys have threatened to do N in. The jeep can travel at 48 km/hr in the sand and at 80 km/hr on the paved road. Can the spy make it in time, or is this the end of N? [*Hint:* The goal is to minimize time, which is distance divided by speed.]

DISTANCE BETWEEN MOVING OBJECTS A truck is 300 miles due east of a car and is traveling west at the constant speed of 30 miles per hour. Meanwhile, the car is going north at the constant speed of 60 miles per hour. At what time will the car and truck be closest to each other? [*Hint:* You will simplify the calculation if you minimize the *square* of the distance between the car and truck rather than the distance itself. Can you explain why this simplification is justified?]

INSTALLATION COST A cable is to be run from a power plant on one side of a river 1,200 meters wide to a factory on the other side, 1,500 meters downstream. The cost of running the cable under the water is \$25 per meter, while the cost over land is \$20 per meter. What is the most economical route over which to run the cable?

INSTALLATION COST Find the most economical route in Exercise 21 if the power plant is 2,000 meters downstream from the factory.

POSTER DESIGN A printer receives an order to produce a rectangular poster containing 648 square centimeters of print surrounded by margins of 2 centimeters on each side and 4 centimeters on the top and bottom. What are the dimensions of the smallest piece of paper that can be used to make the poster? [*Hint:* An unwise choice of variables will make the calculations unnecessarily complicated.]

PACKAGING A cylindrical can is to hold 4π cubic inches of frozen orange juice. The cost per square inch of constructing the metal top and bottom is twice the cost per square inch of constructing the cardboard side. What are the dimensions of the least expensive can?

PACKAGING Use the fact that 12 fluid ounces is approximately 6.89π cubic inches to find the dimensions of the 12-ounce soda can that can be constructed using the least amount of metal. Compare these dimensions with those of one of the soda cans in your refrigerator. What do you think accounts for the difference?

PACKAGING A cylindrical can (with top) is to be constructed using a fixed amount of metal. Use calculus to derive a simple relationship between the radius and height of the can having the greatest volume.

CONSTRUCTION COST A cylindrical container with no top is to be constructed to hold a fixed volume of liquid. The cost of the material used for the bottom is 3 cents per square inch, and that for the curved side is 2 cents per square inch. Use calculus to derive a simple relationship between the radius and height of the least expensive container.

PRODUCTION COST Each machine at a certain factory can produce 50 units per hour. The setup cost is $80 per machine, and the operating cost is $5 per hour. How many machines should be used to produce 8,000 units at the least possible cost? (Remember that the answer should be a whole number.)

COST ANALYSIS It is estimated that the cost of constructing an office building that is n floors high is thousand dollars. How many floors should the building have in order to minimize the average cost per floor? (Remember that your answer should be a whole number.)

INVENTORY An electronics firm uses 600 cases of components each year. Each case costs $1,000. The cost of storing one case for a year is 90 cents, and the ordering fee is $30 per shipment. How many cases should the firm order each time to keep total cost at a minimum? (Assume that the components are used at a constant rate throughout the year and that each shipment arrives just as the preceding shipment is being used up.)

INVENTORY A store expects to sell 800 bottles of perfume this year. The perfume costs $20 per bottle, the ordering fee is $10 per shipment, and the cost of storing the perfume is 40 cents per bottle per year. The perfume is consumed at a constant rate throughout the year, and each shipment arrives just as the preceding shipment is being used up.

a. How many bottles should the store order in each shipment to minimize total cost?

b. How often should the store order the perfume?

INVENTORY A manufacturer of medical monitoring devices uses 36,000 cases of components per year. The ordering cost is $54 per shipment, and the annual cost of storage is $1.20 per case. The components are used at a constant rate throughout the year, and each shipment arrives just as the preceding shipment is being used up. How many cases should be ordered in each shipment in order to minimize total cost?

PRODUCTION COST A plastics firm has received an order from the city recreation department to manufacture 8,000 special Styrofoam kickboards for its summer swimming program. The firm owns 10 machines, each of which can produce 30 kickboards an hour. The cost of setting up the machines to produce the kickboards is $20 per machine. Once the machines have been set up, the operation is fully automated and can be overseen by a single production supervisor earning $15 per hour.

a. How many of the machines should be used to minimize the cost of production?

b. How much will the supervisor earn during the production run if the optimal number of machines is used?

c. How much will it cost to set up the optimal number of machines?

RECYCLING To raise money, a service club has been collecting used bottles that it plans to deliver to a local glass company for recycling. Since the project began 80 days ago, the club has collected 24,000 pounds of glass for which the glass company currently offers 1 cent per pound. However, because bottles are accumulating faster than they can be recycled, the company plans to reduce by 1 cent each day the price it will pay for 100 pounds of used glass. Assume that the club can continue to collect bottles at the same rate and that transportation costs make more than one trip to the glass company unfeasible. What is the most advantageous time for the club to conclude its project and deliver the bottles?

RETAIL SALES A retailer has bought several cases of a certain imported wine. As the wine ages, its value initially increases, but eventually the wine will pass its prime and its value will decrease. Suppose that x years from now, the value of a case will be changing at the rate of $53 - 10x$ dollars per year. Suppose, in addition, that storage rates will remain fixed at \$3 per case per year. When should the retailer sell the wine to obtain the greatest possible profit?

CONSTRUCTION An open box is to be made from a square piece of cardboard, 18 inches by 18 inches, by removing a small square from each corner and folding up the flaps to form the sides. What are the dimensions of the box of greatest volume that can be constructed in this way?

POSTAL REGULATIONS According to postal regulations, the girth plus length of parcels sent by fourth-class mail may not exceed 108 inches. What is the largest possible volume of a rectangular parcel with two square sides that can be sent by fourth-class mail?

POSTAL REGULATIONS Refer to Exercise 37. What is the largest volume of a cylindrical parcel that can be sent by fourth-class mail?

MINIMAL COST A manufacturer finds that in producing x units per day (for $0 \le x \le 100$), three different kinds of cost are involved:
a. A fixed cost of \$1,200 per day in wages
b. A production cost of \$1.20 per day for each unit produced
c. An ordering cost of $\frac{100}{x^2}$ dollars per day. Express the total cost as a function of x and determine the level of production that results in minimal total cost.

TRANSPORTATION COST For speeds between 40 and 65 miles per hour, a truck gets $\frac{480}{x}$ miles per gallon when driven at a constant speed of x miles per hour. Diesel gasoline costs \$3.90 per gallon, and the driver is paid \$19.50 per hour. What is the most economical constant speed between 40 and 65 miles per hour at which to drive the truck?

AVIAN BEHAVIOR Homing pigeons will rarely fly over large bodies of water unless forced to do so, presumably because it requires more energy to maintain altitude in flight in the heavy air over cool water. Suppose a pigeon is released from a boat B floating on a lake 5 miles from a point A on the shore and 13 miles from the pigeon's loft L, as shown in the accompanying figure. Assuming the pigeon requires twice as much energy to fly over water as over land, what path should it follow to minimize the total energy expended in flying from the boat to its loft? Assume the shoreline is straight and describe your path as a line from B to a point P on the shore followed by a line from P to L.

CONSTRUCTION The strength of a rectangular beam is proportional to the product of its width and the square of its depth. Find the dimensions of the strongest beam that can be cut from a wooden log of diameter 15 inches.

CONSTRUCTION The stiffness of a rectangular beam is proportional to the product of its width w and the cube of its depth h. Find the dimensions of the stiffest beam that can be cut from a wooden log of diameter 15 inches. (Note the accompanying figure.)

TRANSPORTATION COST A truck is hired to transport goods from a factory to a warehouse. The driver's wages are figured by the hour and so are inversely proportional to the speed at which the truck is driven. The amount of gasoline used is directly proportional to the speed at which the truck is driven, and the price of gasoline remains constant during the trip. Show that the total cost is smallest at the speed for which the driver's wages are equal to the cost of the gasoline used.

URBAN PLANNING Two industrial plants, A and B, are located 18 miles apart, and each day, respectively emit 80 ppm (parts per million) and 720 ppm of particulate matter. Plant A is surrounded by a restricted area of radius 1 mile, while the restricted area around plant B has a radius of 2 miles. The concentration of particulate matter arriving at any other point Q from each plant decreases with the reciprocal of the distance between that plant and Q. Where should a house be located on a road joining the two plants to minimize the total concentration of particulate matter arriving from both plants?

URBAN PLANNING In Exercise 45, suppose the concentration of particulate matter arriving at a point Q from each plant decreases with the reciprocal of the *square* of the distance between that plant and Q. With this alteration, now where should a house be located to minimize the total concentration of particulate matter arriving from both plants?

OPTIMAL SETUP COST Suppose that at a certain factory, setup cost is directly proportional to the number N of machines used and operating cost is inversely proportional to N. Show that when the total cost is minimal, the setup cost is equal to the operating cost.

AVERAGE PRODUCTIVITY The output Q at a certain factory is a function of the number L of worker-hours of labor that are used. Use calculus to prove that when the average output per workerhour is greatest, the average output is equal to the marginal output per worker-hour. You may assume without proof that the critical point of the average output function is actually the desired absolute maximum. [*Hint:* The marginal output per worker-hour is the derivative of output Q with respect to labor L.]

INSTALLATION COST For the summer, the company installing the cable in Example 3.5.5 has hired Frank Kornercutter as a consultant. Frank, recalling a problem from first-year calculus, asserts that no matter how far downstream the factory is located (beyond 1,200 meters), it would be most economical to have the cable reach the opposite bank 1,200 meters downstream from the power plant. The supervisor, amused by Frank's naivete, replies, "Any fool can see that if the factory is farther away, the cable should reach the opposite bank farther downstream. It's just common sense!" Of course, Frank is no common fool, but is he right? Why?

CONSTRUCTION As part of a construction project, it is necessary to carry a pipe around a corner as shown in the accompanying figure. What is the length of the longest pipe that will fit horizontally?

PRODUCTION COST A manufacturing firm receives an order for q units of a certain commodity. Each of the firm's machines can produce n units per hour. The setup cost is s dollars per machine, and the operating cost is p dollars per hour.
a. Derive a formula for the number of machines that should be used to keep total cost as low as possible.
b. Prove that when the total cost is minimal, the cost of setting up the machines is equal to the cost of operating the machines.

INVENTORY The inventory model analyzed in Example 3.5.7 is not the only such model possible. Suppose a company must supply N units per time period at a uniform rate. Assume that the storage cost per unit is D_1 dollars per time period and that the setup (ordering) cost is D_2 dollars. If production is at a uniform rate of m units per time period (with no items in inventory at the end of each period), it can be shown that the total storage cost is

$$C_1 = \frac{D_1 x}{2}\left(1 - \frac{N}{m}\right)$$

where x is the number of items produced in each run.

a. Show that the total average cost per period is

$$C = \frac{D_1 x}{2}\left(1 - \frac{N}{m}\right) + \frac{D_2 N}{x}$$

b. Find an expression for the number of items that should be produced in each run in order to minimize the total average cost per time period.

c. The optimum quantity found in the inventory problem in Example 3.5.7 is sometimes called the **economic order quantity (EOQ),** while the optimum found in part (b) of this exercise is called the **economic production quantity (EPQ).** Modern inventory management goes far beyond the simple conditions in the EOQ and EPQ models, but elements of these models are still very important. For instance, the just-in-time inventory management described in Example 3.5.7 fits well with the production philosophy of the Japanese. Read an article on Japanese production methods and write a paragraph on why the Japanese regard using space for the storage of materials as undesirable.

EFFECT OF TAXATION ON A MONOPOLY A **monopolist** is a manufacturer who can manipulate the price of a commodity and usually does so with an eye toward maximizing profit. When the government taxes output, the tax effectively becomes an additional cost item, and the monopolist is forced to decide how much of the tax to absorb and how much to pass on to the consumer. Suppose a particular monopolist estimates that when x units are produced, the total cost will be

$$C(x) = \frac{7}{8}x^2 + 5x + 100$$

dollars and the market price of the commodity will be $p(x) = 15 - \frac{3}{8}x$ dollars per unit. Further assume that the government imposes a tax of t dollars on each unit produced.

a. Show that profit is maximized when $x = \frac{2}{5}(10 - t)$.

b. Suppose the government assumes that the monopolist will always act so as to maximize total profit. What value of t should be chosen to guarantee maximum total tax revenue?

c. If the government chooses the optimum rate of taxation found in part (b), how much of this tax will be absorbed by the monopolist and how much will be passed on to the consumer?

d. Read an article on taxation and write a paragraph on how it affects consumer spending.†

ACCOUNT MANAGEMENT Tom requires $10,000 spending money each year, which he takes from his savings account by making N equal withdrawals. Each withdrawal incurs a transaction fee of $8, and money in his account earns interest at the simple interest rate of 4%.

a. The total cost C of managing the account is the transaction cost plus the loss of interest due to withdrawn funds. Express C as a function of N. [*Hint:* You may need the fact that $1 + 2 + \ldots + N = N(N + 1)/2$.]

b. How many withdrawals should Tom make each year in order to minimize the total transaction cost $C(N)$?

Determine where the graph of each of these functions is concave upward and concave downward. Find the x (or t) coordinate of each point of inflection

$f(x) = 3x^5 - 10x^4 + 2x - 5$

Determine all vertical and horizontal asymptotes for the graph of each of these functions.

$$f(x) = \frac{2x - 1}{x + 3}$$

WORKER EFFICIENCY A postal clerk comes to work at 6 A.M. and t hours later has sorted approximately $f(t) = -t^3 + 7t^2 + 200t$ letters. At what time during the period from 6 A.M. to 10 A.M. is the clerk performing at peak efficiency?

MAXIMIZING PROFIT A manufacturer can produce MP3 players at a cost of $90 apiece. It is estimated that if the MP3 players are sold for x dollars apiece, consumers will buy $20(180 - x)$ of them each month. What unit price should the manufacturer charge to maximize profit?

CONCENTRATION OF DRUG The concentration of a drug in a patient's bloodstream t hours after it is injected is given by

$$C(t) = \frac{0.05t}{t^2 + 27}$$

milligrams per cubic centimeter.

a. Sketch the graph of the concentration function.

b. At what time is the concentration decreasing most rapidly?

c. What happens to the concentration in the long run (as $t \to \infty$)?

BACTERIAL POPULATION A bacterial colony is estimated to have a population of

$$P(t) = \frac{15t^2 + 10}{t^3 + 6}$$

t hours after the introduction of a toxin.

a. What is the population at the time the toxin is introduced?

b. When does the largest population occur? What is the largest population?

c. Sketch the graph of the population curve. What happens to the population in the long run (as $t \to \infty$)?

PROFIT A manufacturer can produce sunglasses at a cost of $5 apiece and estimates that if they are sold for x dollars apiece, consumers will buy $100(20 - x)$ sunglasses a day. At what price should the manufacturer sell the sunglasses to maximize profit?

CONSTRUCTION COST A box with a rectangular base is to be constructed of material costing $2/in.^{2} for the sides and bottom and $3/in.^{2} for the top. If the box is to have volume 1,215 in.^{3} and the length of its base is to be twice its width, what dimensions of the box will minimize its cost of construction? What is the minimal cost?

CONSTRUCTION COST A cylindrical container with no top is to be constructed for a fixed amount of money. The cost of the material used for the bottom is 3 cents per square inch, and the cost of the material used for the curved side is 2 cents per square inch. Use calculus to derive a simple relationship between the radius and height of the container having the greatest volume.

REAL ESTATE DEVELOPMENT A real estate developer estimates that if 60 luxury houses are built in a certain area, the average profit will be $47,500 per house. The average profit will decrease by $500 per house for each additional house built in the area. How many houses should the developer build to maximize the total profit? (Remember, the answer must be an integer.)

OPTIMAL DESIGN A farmer wishes to enclose a rectangular pasture with 320 feet of fence. What dimensions give the maximum area if

a. the fence is on all four sides of the pasture?

b. the fence is on three sides of the pasture and the fourth side is bounded by a wall?