1. A projectile is fired at a speed of 840 m/sec at an angle of 60°. How long will it take to get 21 km downrange?

2. Find the muzzle speed of a gun whose maximum range is 24.5 km.

3. A projectile is fired with an initial speed of 500 m1sec at an angle of elevation of 45°.
   a) When and how far away will the projectile strike?
   b) How high overhead will the projectile be when it is 5 km downrange?
   c) What is the highest the projectile will go?

4. A baseball is thrown from the stands 32 ft above the field at an angle of 30° up from horizontal. When and how far away will the ball strike the ground if its initial speed is 32 ft/sec?

5. An athlete throws a 16-lb shot at an angle of 45° to the horizontal from 6.5 ft above the ground at an initial speed of 44 ft/sec. How long after launch and how far from the inner edge of the stopboard does the shot land?

6. A spring gun at ground level fires a golf ball at an angle of 45°. The ball lands 10 m away. What was the ball's initial speed? For the same initial speed, find the two firing angles that make the range 6 m.

7. An electron in a TV tube is beamed horizontally at a speed of 5×10^6 m/sec toward the face of the tube 40 cm away. About how far will the electron drop before it hits?

8. Laboratory tests designed to find how far golf balls of different hardness go when hit with a driver showed that a 100-compression ball hit with a club head speed of 100 mph at a launch angle of 9° carried 248.8 yd. What was the launch speed of the ball? (It was more than 100 mph. At the same time the club head was moving forward, the compressed ball was kicking away from the club face, adding to the ball's forward speed.)

9. A human cannonball is to be fired with an initial speed of $v_o = 80\sqrt{10}/3$ ft/sec. The circus performer (of the right caliber, naturally) hopes to land on a special cushion located 200 ft downrange. The circus is being held in a large room with a flat ceiling 75 ft high. Can the performer be fired to the cushion without striking the ceiling? If so, what should the cannon's angle of elevation be?

10. A golf ball leaves the ground at a 30° angle at a speed of 90 ft/sec. Will it clear the top of a 30-ft tree 135 ft away?

11. A golf ball is hit from the tee to a green elevated 45 ft above the tee with an initial speed of 116 ft/sec at an angle of elevation of 45°. Assuming that the pin, 369 ft downrange, does not get in the way, where will the ball land in relation to the pin?

12. In Moscow in 1987, Natalya Lisouskaya set a women's world record by putting an 8-lb 13-oz shot 73 ft 10 in. Assuming that she launched the shot at a 40° angle to the horizontal 6.5 ft above the ground, what was the shot's initial speed?

13. A baseball hit by a Boston Red Sox player at a 20° angle from 3 ft above the ground just cleared the left end of the "Green Monster," the left-field wall in Fenway Park (Fig. 11.11). This wall is 37 ft high and 315 ft from home plate. About how fast was the ball going? How long did it take the ball to reach the wall ?

14. Show that a projectile fired at an angle of ex degrees, $0 < \alpha < 90$, has the same range as a projectile fired at the same speed at an angle of $(90 - \alpha)$ degrees. (In models that take air resistance into account, this symmetry is lost.)

15. What two angles of elevation will enable a projectile to reach a target 16 km downrange on the same level as the gun if the projectile's initial speed is 400 m/sec?

16. Show that doubling a projectile's initial speed at a given launch angle multiplies its range by 4. By about what percentage should you increase the initial speed to double the height and range?

17. Show that a projectile attains three-quarters of its maximum height in half the time it takes to reach the maximum height.

18. An ideal projectile is launched straight down an inclined plane, as shown in profile in Fig. 11.13.
a) Show that the greatest downhill range is achieved when the initial velocity vector bisects angle AOR.
b) If the projectile were fired uphill instead of down, what launch angle would maximize its range? Give reasons for your answer.

19. In Exercises 1-8, find the curve's unit tangent vector. Also, find the length of the indicated portion of the curve.

20. Find the point on the curve $r(t)=(5\sin t)i + (5\cos t)j + 12t\, l$ at a distance 26π units along the curve from the origin in the direction of increasing arc length.

21. Find the point on the curve $r(t) = (12 \sin t)\, i - (12 \cos t)j + 5t\, k$ at a distance 13π units along the curve from the origin in the direction opposite to the direction of increasing arc length.

22. find the arc length parameter along the curve from the point where $t = 0$ by evaluating the integral

$$s = \int_0^t |\, v(\tau)\, |\, d\tau$$

Then find the length of the indicated portion of the curve.

23. An object of mass m travels along the parabola $y = x^2$ with a constant speed of 10 units/sec. What is the force on the object due to its acceleration at (0, O)? at $(2^{1/2}, 2)$? Write your answers in terms of i and j. (Remember Newton's law, $F = ma$.)

24. Show that the parabola $y = ax^2$, $a \neq 0$, has its largest curvature at its vertex and has no minimum curvature. (Note: Since the curvature of a curve remains the same if the curve is translated or rotated, this result is true for any parabola.)

25. A particle moves in the plane so that its velocity and position vectors are always orthogonal. Show that the particle moves in a circle centered at the origin.

26. Shot put. A shot leaves the thrower's hand 6.5 ft above the ground at a 45° angle at 44 ft/sec. Where is it 3 sec later?

27. Javelin. A javelin leaves the thrower's hand 7 ft above the ground at a 45° angle at 80 fUsec. How high does it go?

28. A golf ball is hit off a tee with a speed of 50 m/s at an angle of 37 degrees above the horizontal. At what two values of x (horizontal distance from the tee) will the ball be 25 m above the ground?

29. Find equations for the osculating, normal, and rectifying planes of the curve $r(t) = ti + t^2 j + t^3 k$ at the point (1, 1, 1).

30. Find parametric equations for the line that is tangent to the curve $r(t) = e^t i + (\sin t) j + \ln(1 - t)k$ at $t = 0$.

31. Find parametric equations for the line tangent to the helix $r(t) = (\sqrt{2}\cos t)\, i + (\sqrt{2}\sin t)\, j + t\, k$ at the point where $t = \pi/4$.

32. A straight river is 100 m wide. A rowboat leaves the far shore at time t = 0. The person in the boat rows at a rate of 20 m/min, always toward the near shore. The velocity of the river at (x, y) is $v = \left(-\frac{1}{250}(y-50)^2 + 10\right) i$ m/min, $0 < y < 100$.

    a) Given that $r(0) = 0\, i + 100\, j$, what is the position of the boat at time t?
    b) How far downstream will the boat land on the near shore?

33. A straight river is 20 m wide. The velocity of the river at (x, y) is $v = -\frac{3x(20-x)}{100} j$ m/min, $0 \le x \le 20$.

    A boat leaves the shore at (0, 0) and travels through the water with a constant velocity. It arrives at the opposite shore at (20, 0). The speed of the boat is always $\sqrt{20}$ m/min.
    a) Find the velocity of the boat.
    b) Find the location of the boat at time t.
    c) Sketch the path of the boat.

34. The line through the origin and the point A (1, 1, 1) is the axis of rotation of a rigid body rotating with a constant angular speed of 3/2 rad/sec. The rotation appears to be clockwise when we look toward the origin from A. Find the velocity v of the point of the body that is at the position B (1, 3, 2).

35. Sketch a typical level surface for the function $f(x,y,z) = x^2 + y^2 + z^2$

36. Find an equation for the level curve of the function f (x, y) that passes through the given point. $f(x,y) = 16 - x^2 - y^2$, $(2\sqrt{2}, \sqrt{2})$.

37. Find all the second order partial derivatives of the function $f(x,y) = x + y + xy$

38. Find the linearization L (x, y) of the function at each point. $f(x,y) = x^2 + y^2 + 1$ at (a) (0,0), (b) (1,1)

39. Suppose T is to be found from the formula $T = x(e^y + e^{-y})$ where x and y are found to be 2 and ln2 with maximum possible errors of $|dx| = 0.1$ and $|dy| = 0.02$. Estimate the maximum possible error in the computed value of T.

40. About how accurately may $V = \pi r^2 h$ be calculated from measurements of rand h that are in error by 1 %?

41. If r = 5.0 cm and h = 12.0 cm to the nearest millimeter, what should we expect the maximum percentage error in calculating $V = \pi r^2 h$ to be?

42. To estimate the volume of a cylinder of radius about 2 m and height about 3 m, about how accurately should the radius and height be measured so that the error in the volume estimate will not exceed 0.1 m^3 ? Assume that the possible error dr in measuring r is equal to the possible error d h in measuring h.

43. Give a reasonable square centered at (1, 1) over which the value of $f(x, y) = x^3y^4$ will not vary by more than ±0.1.

44. Variation in electrical resistance. The resistance R produced by wiring resistors of R_1 and R_2 ohms in parallel (Fig. 12.26) can be calculated from the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

a) Show that

$$dR = \left(\frac{R}{R_1}\right)^2 dR_1 + \left(\frac{R}{R_2}\right)^2 dR_2$$

b) You have designed a two-resistor circuit like the one in Fig. 12.26 to have resistances of $R_1 = 100$ ohms and $R_2 = 400$ ohms, but there is always some variation in manufacturing and the resistors received by your firm will probably not have these exact values. Will the value of R be more sensitive to variation in R_1, or to variation in R_2? Give reasons for your answer.

45. In another circuit like the one in Fig. 12.26, you plan to change R_1 from 20 to 20.1 ohms and R_2 from 25 to 24.9 ohms. By about what percentage will this change R?

46. Error carry-over in coordinate changes
a) If x = 3 ± 0.01 and y = 4 ± 0.01, as shown here, with approximately what accuracy can you calculate the polar coordinates r and θ of the point P (x, y) from the formulas $r^2 = x^2 + y^2$ and $\theta = \tan^{-1}(y/x)$? Express your estimates as percentage changes of the values that r and θ have at the point $(x_o, y_o) = (3,4)$.
b) At the point $(x_o, y_o) = (3,4)$, are the values of r and θ more sensitive to changes in x, or to changes in y? Give reasons for your answer.

47. Estimate how strongly simultaneous errors of 2% in *a*, *b*, and *c* might affect the calculation of the product p(*a*, *b*, *c*) = *abc*.

48. Estimate how much wood it takes to make a hollow rectangular box whose inside measurements are 5 ft long by 3 ft wide by 2 ft deep if the box is made of lumber I/2-in, thick and the box has no top.

49. The area of a triangle is (1/2)ab sin C, where a and b are the lengths of two sides of the triangle and C is the measure of the included angle. In surveying a triangular plot, you have measured a, b, and C to be 150 ft, 200 ft, and 60^o, respectively. By about how much could your area calculation be in error if your values of a and b are off by half a foot each and your measurement of C is off by 2°? See the figure, Remember to use radians.

50. Changing voltage in a circuit. The voltage V in a circuit that satisfies the law V = IR is slowly dropping as the battery wears out. At the same time, the resistance R is increasing as the resistor heats up. Use the equation

$$\frac{dV}{dt} = \frac{\partial V}{\partial I}\frac{dI}{dt} + \frac{\partial V}{\partial R}\frac{dR}{dt}$$

to find how the current is changing at the instant when R = 600 ohms, I = 0.04 amp, dR/dt = 0.5 ohm/sec, and dV/dt = -0.01 volt/sec.

51. Changing dimensions in a box. The lengths a, b, and c of the edges of a rectangular box are changing with time. At the instant in question, a = 1 m, b = 2 m, c = 3 m, da/dt = db/dt = 1 m/sec, and dc/dt = -3 m/sec. At what rates are the box's volume V and surface area S changing at that instant? Are the box's interior diagonals increasing in length, or decreasing?

52. Find the gradient of the function at the given point. Then sketch the gradient together with the level curve that passes through the point.

$f(x, y) = y – x$, (2,1)

53. Find the derivative of the function at P_o in the direction of A.

$f(x,y) = 2xy – 3y^2$, $P_o(5,5)$, A = 4i + 3j

54. Find the directions in which the functions increase and decrease most rapidly at P_o. Then find the derivatives of the functions in these directions.

$f(x,y) = x^2 + xy + y^2$, $P_o(-1,1)$

55. By about how much will g(x, y, z) = x + x cos z - y sin z + y change if the point P (x, y, z) moves from P_o (2, -1, 0) a distance of ds = 0.2 units toward the point P_1 (0, 1, 2)?

56. Find equations for the (a) tangent plane and (b) normal line at the point P_o on the given surface

$x^2 + y^2 + z^2 = 3$, P_o(1, 1, 1)

57. Find an equation for the plane that is tangent to the given surface at the given point.

$z = \ln(x^2 + y^2)$, (1,0,0)

58. Find parametric equations for the line tangent to the curve of intersection of the surfaces at the given point.

Surfaces: $x + y^2 + 2z = 4$, $x = 1$

Point: $(1, 1, 1)$